Carrot and Stick Zoning*

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Abstract

We develop a structural model describing the optimal timing and intensity of capital investment in response to fiscal and regulatory policies. Our model focuses on real estate development and incorporates a government that can simultaneously relax land-use restrictions, offer property tax exemptions, and mandate public good contributions within developments. We calibrate our model to New York City's affordable housing mandate and capture key features of the development process, including the cost and return to height, uncertainty in future returns, and spatial heterogeneity in rents and the built environment. Our results reveal that although both tax exemptions and land-use relaxations increase expected housing production, it is achieved through distinct mechanisms. Tax credits accelerate development by raising the probability of redevelopment, whereas relaxations in land-use increase expected density without accelerating development. We also quantify the public costs associated with each policy, highlighting the trade-offs between expected tax revenue losses and infrastructure costs to support increased density. Finally, we show that allowing voluntary participation in incentive zoning schemes achieves nearly the same level of affordable housing production as mandatory policies but at substantially lower fiscal and infrastructure costs, as developers self-select into programs that best align with market conditions.

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Housing affordability is one of the most salient challenges in urban areas today, particularly in large and economically successful cities (Albouy et al., 2016; Gyourko et al., 2013). In response, many national and local governments have enacted policies to stimulate housing production and expand affordable housing supply.[1] Given the irreversible nature of real estate development, anticipating the effects of such policies requires a model of capital investment with forward-looking developers. This paper provides such a model. It endogenizes the optimal timing and intensity of real estate investment to quantify how public policies that combine regulatory and fiscal incentives influence investment decisions and, thus, the built environment.

In constructing our model, we extend classic models of irreversible investment under uncertainty (Capozza and Li, 1994; McDonald and Siegel, 1986; Paddock et al., 1988). In our model, a real estate developer inherits a developed parcel that provides returns based on its location and existing building. Each parcel is also subject to a specific land-use regulatory regime, where each regime determines the land use that, in turn, defines the return, the volatility, and the maximum allowable density of development. Developers, subject to realistic demolition and construction costs and a non-instantaneous construction period, start construction at the time and intensity that maximizes the present value of their property. This optimal decision rule, however, depends on the regulatory environment. Specifically, we assume that governments can offer property tax exemptions and density bonuses (*Carrots*) to compensate the developer for in-kind provisions of public goods (*Stick*). These public goods may take several forms, including infrastructure, environmental improvements, or social housing provisions.[2] We solve our model ana-

[1] Recent studies have studied the effects of various forms of supply-side policies including upzoning (Büchler and Lutz, 2024; Freemark, 2020, 2023), inclusionary housing mandate with density bonuses (Krimmel and Wang, 2023; Schuetz et al., 2009, 2011), or with fiscal incentives (Baum-Snow and Marion, 2009; Eriksen and Rosenthal, 2010; Soltas, 2022). See Saiz (2023) for a review of various policy options to combat the affordability crisis.

[2] This ability of governments to recapture the value increases resulting from public actions, often referred to as "land value capture" in an urban context (Lincoln Institute of Land Policy, 2022), is a central concept in the current urban policy debate. According to Homsy and Kang (2023), 41% of cities in the United States use some form of incentive zoning. Canadian cities also commonly use development transfers as incentives

lytically, deriving expressions for parcel value, redevelopment probability, and optimal density choices under all policy combination scenarios.

We calibrate our model in three steps using data from the New York City (NYC) real estate market, with a focus on the affordable housing mandate. First, we parametrize and calibrate key model functions that describe the returns to and the cost of real estate development. We allow the return to density to increase with height, reflecting additional utility or production efficiency at higher floors (Ahlfeldt and Barr, 2022; Danton and Himbert, 2018; Liu et al., 2018), and convex construction costs (Ahlfeldt et al., 2023; Ahlfeldt and McMillen, 2018; Eriksen and Orlando, 2022). We calibrate the elasticities with respect to the share of affordable units in a building to reduced-form estimates finding, for instance, that a mandate increase from 10 to 20% reduces property values by 4.36%. Second, we estimate market-specific parameters within each of the NYC neighborhoods. To quantify property values and returns, we combine census data on rents and historical property return data from Costar. We then estimate implied return volatility for each market, matching our model's expected housing production to observed housing construction in each neighborhood. This approach allows us to estimate volatility levels that best align with actual development patterns, following the standard implied volatility estimation for financial options (Black and Scholes, 1973; Dumas et al., 1998). Finally, we account for parcel heterogeneity by calibrating each NYC parcel with its current density, maximum allowable density, estimated obsolescence, and unobserved time-invariant quality, which captures locational advantages such as proximity to transit or amenities.

Before turning to policy counterfactuals, we assess how our baseline calibration predicts key development outcomes, including the probability and intensity of redevelopment for each parcel, the expected number of housing units constructed in each neighbor-

for obtaining privately provided public amenities (Moore, 2013). The German federal legislation obligates developers to compensate for land development by protecting natural areas (Druckenbrod and Beckmann, 2018). Similar policies are expected to be implemented across all European countries under the "No Net Land Take by 2050" policy.

hood, and projected future density and change in property tax revenues. Our baseline estimates of 211,000 forecasted units over a decade closely align with observed construction trends. We observe that development is expected to concentrate in high-demand areas, particularly western Brooklyn and Queens. Because the structural nature of our model explicitly captures the endogenous relation between property values and zoning policies–a link that reduced-form estimates often struggle to capture due to well-documented issues of endogeneity (McMillen and McDonald, 1991; Shertzer et al., 2018)– we observe that existing density, time-invariant quality, and obsolescence play a central role in shaping redevelopment incentives and that lower-density neighborhoods have higher redevelopment option values. Finally, we validate our model's fit by comparing its price predictions to Costar's transaction prices and the NYC Department of Finance assessment values, finding a strong positive correlation.

We then use our model to examine how different combinations of public policies influence the built environment. Specifically, we evaluate three policy instruments and their interactions: a 20% affordable housing mandate, a 25% density bonus, and a 30-year property tax exemption. The results, illustrated in Figure 1, highlight the differential impact of these policies. Panel A shows that both upzoning (Policy [1]) and tax exemption (Policy [2]) policies increase overall housing supply relative to the baseline scenario. We yet observe that they operate through distinct channels. Tax exemptions stimulate development by increasing the probability of parcel redevelopment, whereas density bonuses raise the optimal density of new developments without necessarily accelerating the timing of redevelopment. Adding an affordability mandate with these policies (Policies [4] and [5]) unambiguously reduces the number of expected market-rate units. However, this reduction is offset by an increase in affordable units. Notably, the overall housing supply increases when both incentives are offered together (Policy [6]), though this comes at a cost to municipal finances and infrastructure planning.

Panel B of Figure 1 underscores these potential costs, particularly in terms of expected tax revenue losses and increased density. Tax exemptions (Policies [2] and [5]) significantly reduce the city's fiscal base, potentially constraining the funding of essential public services. In contrast, density bonuses (Policies [1] and [4]) lead to a higher expected density that requires greater investment in transportation, utilities, and public amenities. Combining policies (Policy [6]) yields the largest supply increases but also the largest fiscal and infrastructural burdens. These results, and thus our model, highlight the inherent trade-offs that local governments must balance when designing policies.

We extend our counterfactual analyses in multiple ways. We first explore the heterogeneous effects of these Carrot & Stick policies across different locations, demonstrating that blanket policies produce heterogeneous outcomes across the city. For example, parcels closer to redevelopment thresholds respond more strongly to fiscal incentives, whereas high-quality locations benefit primarily from zoning relaxations. In a last exercise, we incorporate developer incentives by allowing firms to opt into the new regulatory regime or remain under the status quo. Our findings indicate that voluntary adoption delivers nearly the same level of affordable housing production as mandatory policies but at a fraction of the cost. For instance, while mandatory Density Incentive Zoning (Policy [4]) generates 45,633 affordable units, its voluntary counterpart produces 43,384 units while reducing the density increase from 1.26% to just 0.23%. Similarly, while mandatory Fiscal Incentive Zoning (Policy [5]) produces 38,743 affordable units, its voluntary counterpart exceeds this outcome, generating 42,715 units while reducing tax revenue losses from 0.90% to just 0.14%. This efficiency stems from selective developer participation, where only the most viable projects opt in. While only 14.0% of parcels participate in voluntary Density Incentive Zoning, adoption rises to 38.6% when both density and fiscal incentives are bundled (Policy [6]), demonstrating that aligning incentives with developer interests ensures significant participation while minimizing financial inefficiencies and unnecessary density increases. The ability of voluntary policies to achieve affordability gains with minimal fiscal burden highlights the potential for market-driven public-private collaboration, where policy flexibility encourages private developers to meet public affordability goals with minimal strain on city budgets and infrastructure.

Our paper primarily contributes to two strands of literature. First, it contributes to the literature on the determinants of housing and real estate supply, complementing prior work that has emphasized the role of physical and policy constraints (Albouy and Ehrlich, 2018; Baum-Snow and Han, 2024; Glaeser and Gyourko, 2018; Saiz, 2010). Studies such as Ahlfeldt and McMillen (2018), Ahlfeldt and Barr (2022), and Ahlfeldt et al. (2023) highlight the importance of construction technology and the value of height in the emergence of skyscrapers. Eriksen and Rosenthal (2010) find that fiscal subsidies for low-income rental units do not increase supply because they crowd out market-rate developments. However, Soltas (2023) finds that these fiscal subsidies pull investments forward in time.

Our study is more specifically related to the literature encompassing forward-looking agents in dynamic models of real estate supply, including Murphy (2018), who models landowners' strategic timing of construction based on expected price and cost trends, and Favilukis et al. (2023), who analyze affordability policies within a dynamic spatial equilibrium framework. Like these studies, we incorporate forward-looking developers who respond to market conditions and regulatory constraints. However, we extend this work by explicitly modeling the interaction between fiscal and zoning incentives, allowing developers to endogenously choose both the timing and intensity of construction. Unlike Murphy (2018), who focuses on supply inelasticity due to market timing, our model evaluates policy interventions that alter zoning and tax incentives to shape development decisions. Similarly, while Favilukis et al. (2023) emphasize the welfare effects of affordability policies such as rent stabilization and housing vouchers, our study examines incentive-based zoning mechanisms that balance affordability with fiscal sustainability. We also contribute by quantifying the fiscal trade-offs associated with different regulatory regimes, a dimension largely absent in prior work. Unlike these studies, which primarily analyze market responses to exogenous constraints, our model evaluates the effectiveness of voluntary incentive zoning as an alternative to mandatory regulations. By doing so, we provide a framework that integrates private developer incentives with public policy goals, offering new insights into the optimal design of affordability policies.[3] Building specifically on the real options development framework of Lebret and Liu (2023), we extend this class of models that allows for the structural evaluation of policy counterfactual.

Second, our study complements our understanding of the effects of land-use regulations (Gyourko and Molloy, 2015; Quigley and Rosenthal, 2005), that ultimately impacts the urban environment (Glaeser and Gyourko, 2018) and the economy (Duranton and Puga, 2023; Hsieh and Moretti, 2019).[4] For instance, Brueckner and Sridhar (2012) shows that density restrictions in Indian cities reduce the size of cities and residents' welfare, and (Brueckner and Singh, 2020) shows that more stringent regulations increase land prices in U.S. cities. More recently, Büchler and Lutz (2024) shows that upzoning has led to an increase in housing supply of 9% in Zurich, but that it takes several years to materialize. Bertrand and Kramarz (2002) and Manville et al. (2023) show how discretionary zoning regulations delay development, highlighting the importance of the regulatory environment for understanding the timing of development. Krimmel (2021) shows that

[3]Theoretically we also contribute to the long history of real options models (McDonald and Siegel, 1986; Myers, 1977) used to characterize the real estate development decision (Arnott and Lewis, 1979; Capozza and Li, 1994; Grenadier, 1996; Quigg, 1993; Titman, 1985). These studies recognize that uncertainty acts as a deterrent to real estate development (Bulan et al., 2009; Cunningham, 2006), competition increases the exercise of the development option (Grenadier, 2002), and that the redeployment to another real estate use is also a valuable option (Benmelech et al., 2005; Büchler et al., 2023).

[4]Our study also relates to the literature on the causes of zoning and land-use restrictions (Clingermayer, 1993; Duranton and Puga, 2023; Fischel, 2002; Helpman and Pines, 1977; Hilber and Robert-Nicoud, 2013; Hsieh and Moretti, 2019; McMillen and McDonald, 2002; Stull, 1974) by presenting an alternative rationale for implementing strict ex-ante land-use regulations. These regulations enable regulators to engage in subsequent Carrot & Stick negotiations with private stakeholders for the provision of public goods. Although urban planners and political scientists have investigated this mechanism (Freemark, 2020; Homsy and Kang, 2023; Kim, 2020; Korngold, 2022; Moore, 2013), no paper to our knowledge provides an economic analysis of this tool.

the affordability constraint attached to upzoning in Seattle has led to a development increase outside of the designated areas, questioning the balance between incentives and constraints. Our paper adds to this literature by providing a framework that can encompass combinations of fiscal and regulatory policies on development and city costs.

Our study proceeds as follows. In Section, 1, we describe our structural model highlighting developers' decision rules and the city's policy instruments. In Section 2, we provide numerical analysis results of a representative parcel, highlighting the behavior of the model. In Section 3, we describe the parametrization and the empirical setting that we use to calibrate the model. In Section 4, we describe the baseline results of our models and the aggregate outcomes of interest. In Section 5, we provide the counterfactual policy analysis results. In Section 6, we conclude.

1 A Real Estate Development Model with Policy Instruments

In constructing our model, we explicitly recognize that government has regulatory power over urban parcels. For each parcel, the government considers transitioning from current zoning with regulatory environment $\mathcal{P}_0$ to a different regulatory environment $\mathcal{P}$. This new environment $\mathcal{P}$ that might include changes in land use, variation in density (ϕ), a mandate to include public goods on the site (α), and a tax exemption on completed buildings of τ years[5]. Because private developers factor each policy instrument into the timing and intensity of their building construction, we incorporate these policies into our structural model of real estate development. We first describe the developers' optimal decision rules and then lay out the mechanisms through which each policy instrument affects the built environment.

[5] It follows that $\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$ and zoning $\mathcal{P} \neq \mathcal{P}_0$.

1.1 The Real Estate Developer Decision Rule

We build upon the stochastic financial models of Merton (1974) and Black and Cox (1976), with a particular focus on the timing of investment decisions as explored in Majd and Pindyck (1987). We assume that (i) the public and private sectors have access to complete information, (ii) properties are continuously traded within arbitrage-free and complete markets, (iii) both public and private sectors have access to borrowing and lending at a risk-free rate r, (iv) private developers aim to maximize wealth, and (v) the asset value remains independent of capital structure choices.[6]

We further assume that the market-rate value $(V_t^0)_{t\geq 0}$ expressed as per unit of density under zoning $\mathcal{P}_0$ is governed, under the risk-neutral measure $\mathbb{Q}$, by:

$$dV_t^0 = (r - \delta_0)V_t^0 d_t + \sigma_0 V_t^0 dW_t \tag{1}$$

where δ_0 and σ_0 are, respectively, the unlevered before-tax payout rate and the constant volatility under zoning $\mathcal{P}_0$, and $(W_t)_{t\geq 0}$ is a standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. We thus allow the stochastic process describing real estate demand to vary by zoning, defined by land use (or a bundle of uses), which allows for the mixture of land uses on the same plot of land to reduce σ_0 through diversification. Denoting V as the value at $t = 0$, we can rewrite the value of a market rent per unit of density at $t \geq 0$ as $V_t^0 = Ve^{(r-\delta_0-\frac{\sigma_0^2}{2})t+\sigma_0 W_t} = Ve^{\sigma_0 Z_t^0}$, where $Z_t^0 = b_0 t + W_t$, and $b_0 = \frac{1}{\sigma_0}(r - \delta_0 - \frac{\sigma_0^2}{2})$.

Before discussing development optionality, we discuss two adjustments to obtain the market value of a property that we denote $\pi_0(V, F_B, o_B)$. First, we assume that the government imposes a tax proportional to the rental value of the developed properties,

[6] Implicitly, our model also assumes that the amount of supply in the market does not impact the asset value of the underlying property. Although this assumption is innocuous in the case of a price-taker individual developer, it might create bias when the model applies to the entire city in the sort of counterfactual analyses performed. We discuss in more detail this potential issue in Section 5.5.

which effectively reduces the payout return to the developer by ψ. Second, we define $\Gamma(F, o, \alpha)$, the function that, based on the characteristics of the built property, scales the market value of a unit of density V_t to yield the stochastic rental value of a developed property. In particular, this value depends on the property density (F), the property obsolescence rate (o) and the share of built space dedicated to public goods (α). For example, $\Gamma(1, 0, 0) = 1$ corresponds to a property with a density of 1 ($F_B = 1$), with no operational efficiency losses due to obsolescence ($o = 0$), and no mandate to include public goods on the site ($\alpha = 0$). We revert to the parametrization of $\Gamma(F, o, \alpha)$ in Section 3. Consequently, the value of the developed property without its development option can be expressed as

$$\pi_0(V, F_B; \Theta) = \mathbf{E}_{\mathbb{Q}} \left\{ \int_0^{\infty} e^{-ru} \left(\delta_0 - \psi\right) \Gamma(F_B, o_B, 0) V_u^0 \mathrm{d}u \right\} \tag{2}$$

where Θ is the vector of parameters.

Development decision under current regulatory environment. Private developers have the option to replace the current building with a new one constructed at density $F \leq \bar{F}_0$. Exercising this option entails an instantaneous demolition cost $\gamma(F_B)$, and an annual construction cost during the construction period of d years, denoted as $\kappa(F, \alpha)$, which depends on the density of the new construction F, and the type of asset built.

In this framework, developers exercise the option to expand their property under $\mathcal{P}_0$ only when favorable market conditions prevail. We denote V_0^e, the predetermined level (i.e., expansion barrier) at which they initiate construction of a building of height F, and $T_{V_0^e}$, the time this barrier is reached for the first time. Hence, to complete the property development at density F, developers need a robust market that persists for d years. We denote $H^+_{V_0^e,d}$, the *Parisian* time at which the property is fully developed.[7]

[7]Formally these stopping times are defined as $T_{V_0^e}(W) = \inf\{t \geq 0 : W_t = V_0^e\}$, and $H^+_{V_0^e,d}(W) = \inf\{t \geq 0 : (t - g_t^{V_0^e}(W)) \geq d, W_t \geq V_0^e\}$ with $g_t^{V_0^e}(W) = \sup\{s \leq t : W_s = V_0^e\}$ (Chesney et al., 1997).

Given the financial market assumptions, the two fundamental theorems of asset pricing ensure the existence of a unique probability measure $\mathbb{Q}$, under which the price of any asset, discounted at the risk-free rate, behaves as a martingale. As a result, we can express the value of the property under $\mathcal{P}_0$ inclusive of the development option as follows:

$$\pi_0(V, F_B, F; \Theta) = \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_{V_0^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0) V_u^0 \mathbf{1}_{V_u^0 < V_0^e}\, \mathrm{d}u\right\}}_{\text{Value before development}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{e^{-rT_{V_0^e}}\gamma(F_B)\right\}}_{\text{Demolition costs}}$$
$$- \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V_0^e,d}} e^{-ru}\kappa(F,\alpha)\mathbf{1}_{V_u^0 > V_0^e}\, \mathrm{d}u\right\}}_{\text{Development costs}} + \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V_0^e,d}}^{\infty} e^{-ru}\,(\delta_0 - \psi)\,\Gamma(F, 0, \alpha) V_u^0 \mathrm{d}u\right\}}_{\text{Value after development}} \quad (3)$$

where $\mathbf{1}_x$ represents the indicator function for the event x. Equation (3) consists of the sum of four present-value terms, which respectively represent the cash flows generated by the built property until construction starts, demolition cost, costs incurred during the property's construction period, and the cash flows obtained after the property is developed. Before solving this value function analytically in Proposition 1 below, we first discuss how we incorporate the different policy instruments into this value function.

Adding policy instruments into the development decision rule. We model three distinct policy instruments that local governments can use independently or together. First, the government can mandate that α percent of the development be allocated to the public. These public goods have a payout rate δ_α, constant volatility of asset return σ_α, and covariance with the market's rent process of $\sigma_{\alpha,0}$. As discussed above, increasing α also impacts the value of the property and the construction cost through $\Gamma(.)$ and $\kappa(.)$, respectively. The market-rate value per unit of density under the new regulatory regime $\mathcal{P}$ is thus governed by the process

$$dV_t = (r - \delta)\, V_t d_t + \sigma V_t dW_t \tag{4}$$

where we assume that $\delta = \alpha\delta_\alpha + (1-\alpha)\delta_0$ and $\sigma^2 = \alpha^2\sigma_\alpha^2 + (1-\alpha)^2\sigma_0^2 + 2\alpha(1-\alpha)\sigma_{\alpha,0}$.[8] Second, the government can provide additional density bonuses by allowing an additional height ϕ, so the developer chooses F provided that $F \leq \bar{F}$ where $\bar{F} = \bar{F}_0(1+\phi)$. Finally, developers can receive tax exemptions for τ years once construction is completed. The policy instrument vector is thus $\mathcal{P} = \{\alpha, \phi, \tau\}$.

The value of a property under the new regulatory environment is expressed as:

$$\begin{aligned}
\pi(V, F_B, F, \alpha, \phi, \tau; \Theta) = {} & \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{T_{V^e}} e^{-ru}\ (\delta_0 - \psi)\ \Gamma(F_B, o_B, 0) V_u\, \mathbf{1}_{V_u < V^e}\, \mathrm{d}u \right\}}_{\text{Value before completed development}} \\
& - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ e^{-rT_{V^e}} \gamma(F_B) \right\}}_{\text{Demolition costs}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru} \kappa(F,\alpha)\, \mathbf{1}_{V_u > V^e}\, \mathrm{d}u \right\}}_{\text{Development costs}} \\
& + \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}}^{\infty} e^{-ru} \delta\, \Gamma(F, 0, \alpha) V_u\, \mathrm{d}u \right\}}_{\text{Value after development gross of tax}} - \underbrace{\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru} \psi\, \Gamma(F, 0, \alpha) V_u\, \mathrm{d}u \right\}}_{\text{Property taxes after } \tau \text{ years of exemption}}
\end{aligned} \tag{5}$$

Similarly to Equation (3), the property value in $\mathcal{P}$ consists of the sum of present values with an additional final term that captures the impact of postponing property taxes for τ years after the development has been completed.

[8]The sign and magnitude of the covariance $\sigma_{\alpha,0}$ give rise to several scenarios. If $\sigma_{\alpha,0} < 0$, the public good provides risk reduction through diversification benefits, potentially enhancing the property's value due to *within-building* positive externalities. This scenario could occur, for instance, with investments in infrastructure that improve access to public transportation. Conversely, if $\sigma_{\alpha,0} > 0$, the public good amplifies risk, which may negatively impact the property's value. An example of this could be the introduction of affordable housing, where well-documented *within-building* negative externalities, such as NIMBY (Not In My Backyard) concerns, may decrease property values.

1.2 Analytical Solutions

To derive analytical solutions to (5), we modify the value before development is completed (first term in Equation [5]) as follows

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{T_{V^e}} e^{-ru}\,(\delta_0-\psi)\,\Gamma(F_B,o_B,0)\,V\,\mathbf{1}_{V_u<V^e}\mathrm{d}u \right\}$$

assuming that the rental income $(\delta_0-\psi)\,\Gamma(F_B,o_B,0)\,V$ remains constant until the demolition decision.

Proposition 1 *In the presence of the expansion threshold V^e, the property value $\pi(.)$ satisfies*

$$\begin{aligned}
&\pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta)= \\
&\qquad \underbrace{\Gamma(F_B,o_B,0)\frac{(\delta_0-\psi)\;V}{r}\left[1+\left[\frac{r}{\lambda(b-\lambda)}-1\right]\left(\frac{V^e}{V}\right)^{\xi}\right]}_{\textit{Value before completed development}}-\underbrace{\gamma(F_B)\left(\frac{V^e}{V}\right)^{\xi}}_{\textit{Demolition costs}} \\
&-\underbrace{\frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\left(\frac{V^e}{V}\right)^{\xi}}_{\textit{Development costs}}+\underbrace{\Gamma(F,0,\alpha)V^eC(d)\left(\frac{V^e}{V}\right)^{\xi}}_{\textit{Value after development}}-\underbrace{\Gamma(F,0,\alpha)\frac{\psi h}{\delta}V^eC(d)\left(\frac{V^e}{V}\right)^{\xi}}_{\textit{Property taxes after } \tau \textit{ years of tax credits}}
\end{aligned} \tag{6}$$

with $b=(1/\sigma)(r-\delta-\sigma^2/2)$, $\lambda=\sqrt{2r+b^2}$, $h=e^{-\left[(b+\sigma)^2+\lambda^2\right]\frac{\tau}{2}}$ *and* $\xi=(1/\sigma)(b-\lambda)$ *and where also*

$$A(d)=\tfrac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})},\; B(d)=\tfrac{b+\lambda}{2\lambda}-\tfrac{b-\lambda}{2\lambda}\tfrac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})},\; \textit{and}\quad C(d)=\frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}$$

with $\Phi(x)=\int_0^{+\infty} ze^{zx-\frac{z^2}{2}}\,\mathrm{d}z=1+x\sqrt{2\pi}e^{\frac{x^2}{2}}\mathcal{N}(x)$, $\mathcal{N}(.)$ *being the standard normal cumulative distribution function.*

The proof can be found in appendix B.1.

Although explicitly controlled by developers, the timing of development is endogenous to the vector of policy choices $\mathcal{P}$. Conditional on the current market and policy conditions, developers determine the most favorable time to initiate construction based on the stochastic process reaching a specific threshold V^{e*}.

Proposition 2 *When developers select the optimal timing for development under regulatory regime $\mathcal{P}$, the expansion threshold is given by*

$$V^{e*} = \frac{\xi}{1+\xi}\frac{1}{\vartheta}\left[\gamma(F_B) + \frac{\kappa(F,\alpha)}{r}\Big(B(d) - A(d)\Big) - \Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right]\right] \quad (7)$$

with $\vartheta = \Gamma(F, 0, \alpha)\left(1 - \frac{\psi h}{\delta}\right)C(d)$.

The proof can be found in appendix B.2.

From Proposition 2, we note that the expansion barrier increases, and thus, developers delay development when construction and demolition costs rise. Recalling that $T^{V^{e*}}(V) = \inf\{t \geq 0 | V_t = V^{e*}\}$ represents the first time the value of an unlevered unit of density reaches the expansion threshold, we can derive the probability that development begins before time t.

Proposition 3 *Given the expansion threshold V^{e*}, the probability that development starts before time t under regulatory regime $\mathcal{P}$ satisfies*

$$\begin{aligned} \mathbf{P}\left(T^{V^{e*}} \leq t\right) &= \int_0^t \mathbf{P}\left(T^{V^{e*}} \in \mathrm{d}s\right) \\ &= \int_0^t \frac{a}{\sqrt{2\pi s^3}} \exp\left(-\frac{a^2}{2s}\right) \mathrm{d}s \end{aligned} \quad (8)$$

where $a = \ln(V^{e*}/V)/\sigma$ and $\mathbf{P}(.)$ is the probability under the new measure $\mathbb{P}$, such that $(Z_t)_{t\geq 0}$[9] *becomes a Brownian motion under $\mathbb{P}$.*

[9] $Z_t = bt + W_t$

The proof can be found in appendix B.3

1.3 Optimal Density

In the analytical solutions presented above, developers chose an optimal timing of construction conditional on construction density $F \leq \bar{F}$. We numerically derive F^*, the optimal density, by maximizing the value of the property subject to the maximum allowable density and optimal barrier V^{e*}:

$$\begin{aligned} F^* = \arg\max_{F} \quad & \pi(V, V^{e*}, F_B, F, \alpha, \phi, \tau; \Theta) \\ \text{s.t.} \quad & 0 \leq F \leq \bar{F} \end{aligned} \tag{9}$$

This numerical optimization allows, in turn, to derive the property value $\pi^*(.)$, the optimal barrier V^{e**}, and development probability $\mathbf{P}^*$ that corresponds to the developer's decision rule considering both the optimal timing and intensity of development.

The general solutions in Equations (6), (7) and (8) combined with the maximization function over density F in equation (9) provide expressions for the property value and the probability of development under a flexible policy framework. We can now evaluate all policy outcomes embedded in this setup. For example, we can derive the property value and development probability under commonly used regulatory environments such as (1) current zoning $\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$, (2) upzoning $\mathcal{P} = \{\alpha = 0, \phi > 0, \tau = 0\}$, or (3) mandatory inclusionary programs with density bonuses $\mathcal{P} = \{\alpha > 0, \phi > 0, \tau = 0\}$, or with tax incentives $\mathcal{P} = \{\alpha > 0, \phi = 0, \tau > 0\}$. Beyond these three policy instruments at the core of our study, we can also study changes from one land use (e.g., office) to another land use (e.g., residential), if we modify the underlying stochastic process as well as the functions $\Gamma()$ and $\kappa()$ in addition to the other changes implied by a change of $\mathcal{P}$.

2 Parcel-level numerical analysis

Before turning to the analysis of policies from the city perspective, we first provide evidence that our model captures the trade-off between carrots (ϕ and τ) and sticks (α) both in the valuation of the parcel and the probability of development. We use a representative parcel close to redevelopment to create those comparative statics that we relax below when incorporating heterogeneous city parcels. We use the parametrization and the calibration described in more detail in Section 3. In short, this calibration not only captures return and cost to density but also considers a mandate α that reduces the value, the construction cost, the return, and the return volatility

We first show in Figure 3 the impact of each policy instrument on property values (left panels) and the probability of development within 10 years (right panels). Figure 3 also illustrates how these effects vary when initial demand is higher or lower than the baseline. Figure A1 further separates the effects of the policy instruments on the property value by highlighting the effects on the value of the developed property and the value of the redevelopment option (i.e., the final four terms in Equation [6]).[10] Table 2 shows the sensitivity of these results with respect to key elasticities.

Affordable Housing Mandates (α). Panel (a) of Figure 3 shows that increasing the affordable unit requirement consistently decreases both property values and development probabilities. For instance, Table 2 indicates that a 20% mandate leads to a 5.22% drop in property values and a 7.97 percentage point reduction in the probability of development. Figure A1 confirms that the loss in property value from increasing α is due to a reduction in the redevelopment value. This lower development probability increases the present value of the existing building, given the longer expected cash flows.

In Table 2, we see that the same 20% mandate in parcels with higher demand (V up

[10] Marginal effects with respect to other parameters are illustrated in Figure A2.

by 100%) causes a larger decline in property value (8.96%) and a sharper drop in development probability (8.23 percentage points). This result illustrates the greater impact that affordable housing mandates have in high-demanded areas. If the rental discount is doubled for affordable units (ϵ_α) then the negative effects on property value (-6.11%) and 10-years development probability (-10.87 percentage points) are larger. Finally, if affordable housing development costs are 100% higher, the 20% mandate would decrease property values by -4.84% and reduce the likelihood of development (-6.42 percentage points). This sensitivity analysis demonstrates that cost escalation can make development less feasible with a higher negative impact on property value.

Density Bonuses (ϕ). Panel (b) of Figure 3 shows that increasing the maximum allowed density increases property value. However, it can also reduce the probability of development at high values of ϕ. This numerical analysis clearly indicates that property values and development probabilities are not necessarily in sync with one another. Increasing ϕ raises construction costs through $\kappa()$ but also boosts income through greater density. Because developers can exercise the option to build at higher density or not, higher density always increases property value. However, for high values of ϕ, developers may delay construction, strategically reducing the observed development probability. Thus, the higher property values result from the increased present value of both the developed property and the redevelopment option (Figure A1).

Table 2 shows that a 50% increase in allowable density (setting $\phi = 0.5$) increases property values by 7.09% and development probabilities by 1.87 percentage points. If the increase is in high-demand areas, this same density bonus magnifies property values. A property value increase of 24.14% and development probability increase of 12.06 percentage points is realized when areas are targeted that have twice the underlying rental value. Increasing the marginal benefits of height (ϵ_F) shows that a $\phi = 0.5$ marginally raises the property value and development probability by 9.65% and 3.34 percentage points, respectively, compared to our baseline case. Although a consensus exists on the average height elasticity of construction cost, the elasticity also depends on the bedrock depth, which impacts the elasticity by factors of 10 (Ahlfeldt et al., 2023). We show that increasing the height elasticity of construction costs by only 50% results in a modest property value increase (0.34%) and a negative impact on development probability (-4.04 percentage points). Consequently, this result highlights the role of construction technology in the real estate development industry.

Tax Exemptions (τ). Offering tax exemptions unambiguously increases property value and accelerates development (Panel c of Figure 3). The increase in property value comes solely from the redevelopment option, as the tax exemption applies to the newly developed building (Panel c of Figure A1). For example, offering a 20-year tax credit raises property value by 4.58% and increases development probability by 7.66 percentage points for this representative parcel. Offering identical tax exemption to high-demand parcels (doubling V) boosts property value by 9.31% and raises development probability substantially (+9.88 percentage points). As expected, higher property tax rates amplify the benefits of tax credits. Doubling the property tax rate, for example, increases the impact of a 20-year tax credit, raising property value and development probability compared to the baseline case.

3 Model Calibration

The calibration of our model focuses on the affordable housing mandate, α, in residential zoning, using NYC as a case study. The calibration process is conducted in two stages. First, we define the functional forms of the three key functions embedded in the model—$\Gamma(.)$, $\gamma(.)$, and $\kappa(.)$—and estimate their respective elasticities using either micro-based evidence or values from the existing literature. Second, we incorporate market-specific parameters and parcel-level heterogeneity to calibrate the model for the entire

NYC. A detailed description of all parameters is provided in Table 1.

3.1 Model Parametrization

3.1.1 The Density and Mandated Public Good Share Elasticities of Value (ϵ_F and ϵ_α).

In our model, the underlying market value of an unlevered unit of density V is scaled by the function $\Gamma()$ to reflect the built space of the parcel. We parametrize this scaling function as

$$\Gamma(F, o, \alpha) = F^{(1+\epsilon_F)}(1-\alpha)^{\epsilon_\alpha}(1-o) \tag{10}$$

where $\epsilon_F > 0$ is the density elasticity of market-rate value, and $\epsilon_\alpha > 0$ denotes the non-market-unit share elasticity of value. We calibrate ϵ_F to 0.07 following the results in the literature on the economics of skyscrapers (Ahlfeldt and Barr, 2022; Ahlfeldt et al., 2023; Danton and Himbert, 2018).[11] This positive ϵ_F captures the amenity value associated with better views and lower noise at higher floors, which translates into increased market-rate value.

Given our focus on affordable housing mandates in the counterfactual analysis and the lack of existing elasticities, we use reduced-form estimates to calibrate the value elasticity with respect to α. Rearranging Equation (10) and assuming that a unit of density (F) is an apartment unit, one can estimate ϵ_α by regressing the log price per apartment (PPU_i) on $(\log(1 - AffShare_i))$

$$\log PPU_i = \beta + \epsilon_F \log Unit_i + \epsilon_\alpha \log\left(1 - AffShare_i\right) + \nu_i \tag{11}$$

where $\beta \approx \log(V)$.[12] We can estimate this regression with real estate transactions using

[11] Using the elasticity of rent instead of value presents no issues unless the capitalization rate varies by building height, which could happen if there is a risk or growth rate differential between taller and shorter buildings. We also abstract from the value increase associated with ground-floor properties (Danton and Himbert, 2018; Liu et al., 2018), since our model focuses on additional floor space ϕ, which inherently refers to construction above the ground floor.

[12] This rearrangement implicitly assumes that the observed building prices do not incorporate the option

ordinary least squares (OLS) if $(1 - AffShare_i)$ is randomly assigned to buildings. Because it is highly unlikely that the allocation of affordable units in buildings is randomly assigned, we limit the transactions to properties that include both affordable and market-rate units, thus concentrating on the extensive margin of variation in α. We merged the transaction of apartment buildings from Costar with the list of properties having affordable units from the Affordable Housing Production by Building dataset provided by the NYC Department of Housing Preservation and Development. To control for the impact of location and the timing of sales on apartment prices, we include borough and year fixed effects in our regression, in addition to the log of apartment units.[13] The results in Table A1 indicate that the elasticity of price with respect to $(1 - \alpha)$ ranges from 0.198 to 0.273.[14] To contextualize these findings, an ϵ_α of 0.2 is equivalent to a rental discount on affordable units compared to market-rate units ranging from 20.4%-25.9%, under reasonable discount rate assumptions.[15] We also observe heterogeneity across boroughs, with the lowest and highest discount occurring in the Bronx and in Manhattan, respectively.

3.1.2 The demolition cost function.

To account for the expenses during the demolition phase, we parametrize the demolition cost function as

$$\gamma(F) = k_d F^{\eta_D} \tag{12}$$

Here, $0 < \eta_D < 1$ represents the density elasticity of demolition costs, while k_d is a cost shifter associated with demolishing one unit of density. We calibrate these parameters based on insights from discussions with real estate developers. Specifically, developers

value of redevelopment. To align with this assumption, we focus on transactions involving buildings less than 50 years old, as these are likely to have negligible redevelopment option value.

[13] Because some years contain only one transaction, we create fixed effects that capture the various periods of the recent economic cycles.

[14] Although not the primary focus of this estimation, we note that the ϵ_F values in Table A1 align with estimates from the existing literature.

[15] We recognize that the level of rental discount associated with affordable housing is likely within the policy choice set $\mathcal{P}$. To shed light on how this can change the incentive of developers, we provide an evaluation of how our main results change with respect to this parameter in Section 5 below.

noted that fixed costs in demolition projects typically account for 50-60% of total demolition costs, with the remainder attributed to the structure itself. Additionally, developers indicated that demolition costs generally constitute 13-20% of total development costs, including both demolition and construction. The first statistic is matched to the mean fixed costs predicted by our model resulting in a calibration of $\eta_D = 0.72$. The second statistic is used to establish the relation between k_d and k_c, as discussed in the next section. [16]

3.1.3 The construction cost function.

To account for construction costs that are convex in height as well as the potential differences in costs associated with increasing α, we define the annual cost of construction as

$$\kappa(F, \alpha, d) = \frac{1}{d} k_c F^{(1+\eta_F)}(1 - \alpha\, \eta_\alpha) \tag{13}$$

where k_c is a scalar capturing the baseline cost of construction of a single unit of density, $\eta_F > 1$ is the density elasticity of construction cost, and η_α captures the discount/premium in construction cost for space dedicated to the public good α. We use the average estimates from the literature to calibrate the height elasticity of construction cost. For instance, Ahlfeldt and McMillen (2018) estimate a 0.61 height elasticity of cost of construction for residential buildings that have more than 5 floors, Ahlfeldt et al. (2023) calibrate their residential height elasticity of construction cost at 0.55, and Ahlfeldt and Barr (2022) estimate an elasticity of 0.56 for tall residential buildings. We thus set $\eta_F = 0.60$ to reflect increases in construction cost due to height.

The second elasticity, η_α, reflects the cost reduction resulting from changes in the quality of fixtures and finishes associated with non-market units. To calibrate η_α, we rely on data from the National Apartment Association's Income and Expenses Report as

[16] We observe that the probability of development and the property value are homogeneous functions of degree zero and one, respectively, with respect to the level parameters V, k_d, and k_c. As a result, their relations can be expressed in relative terms rather than absolute levels without affecting the validity of our counterfactual analysis.

cited in the RCLO report on The Affordable Housing Asset Class. This report offers detailed insights into capital expenditures per square foot for market-rate and subsidized housing units from 2012 to 2021. On average, subsidized units receive 19.9% less capital expenditure per square foot compared to market-rate units. Acknowledging that capital expenditures differ from construction costs, we assume that the variation in capital intensity serves as a reasonable proxy for the construction cost differential. Based on this, we set $\eta_\alpha = 0.199$.

Construction length. We calibrate the length of the construction period using data from the DCP Housing Database Project-Level Files. We compute the duration from permit issuance to building completion. We restrict the sample to muti-family unit buildings that private for-profit developers construct. As the baseline calibration for d we use 2.44 years, the median time between permit issuance to building completion.[17]

Importance of non-instantaneous construction duration. The impact of uncertainty on investment behavior changes fundamentally when construction is non-instantaneous. In the classical real options framework with instantaneous investment (Capozza and Li, 1994), uncertainty always delays investment by raising the expansion barrier. Figure 2, Panel A, confirms this result for short construction durations (d approaching zero), where greater uncertainty increases V^{e*}, meaning developers require a higher expected return before initiating construction. However, for longer d, the effect of uncertainty is non-monotonic—in low-volatility markets, increasing d lowers the expansion barrier, allowing earlier investment. This divergence from (Capozza and Li, 1994) occurs because, with a longer construction period, developers consider not only when to invest but whether conditions will remain favorable throughout construction.

[17]Through informal discussions with developers, we learned that the size of a building surprisingly has only a minor impact on construction duration. For instance, iconic projects such as the The Empire State Building were completed in just 1 year and 45 days, highlighting that factors other than scale often dominate construction timelines.

Figure 2, Panel B, shows a second key contrast: the probability of development rises with uncertainty, as in classical models, but the threshold effect is stronger when d is long. For short d, higher uncertainty steadily increases development probability, since future prices are more likely to reach profitable levels. However, for long d, developers hesitate at moderate uncertainty levels, waiting for clearer signals, but once uncertainty is high enough, the probability of investment rises sharply. This pattern differs from the *incremental* investment framework in Bar-Ilan and Strange (1999), where uncertainty lowers investment intensity at every stage by discouraging the addition of capital. Unlike *lumpy* investment, where uncertainty affects the timing and size of a one-time decision, in *incremental* investment, firms make many small decisions over time, and each of those decisions is negatively impacted by uncertainty. This continuous adjustment means that investment is always scaled down in response to uncertainty, whereas in *lumpy and lengthy* investment, as is the case here, uncertainty alters the timing but does not reduce intensity once investment occurs. This analysis highlights a key limitation of models that assume instantaneous construction or incremental investments. Such models would tend to overestimate the likelihood of development, leading to inflated forecasts of new housing supply. Our model yields more realistic estimates of expected development outcomes, enhancing the accuracy of policy evaluations.

Baseline construction and demolition costs There are three parameters that govern the levels of our model, namely the value of a unit of non-obsolete density, the demolition cost k_d, and construction costs k_c that are all expressed per unit of density. Because the probability of development and the property value are homogeneous functions of degree zero and one, respectively, with respect to these, we set V to outside data (see following section) that will vary across parcels and calibrate the other two using the method of moments. We assume that they are constant across NYC neighborhoods given that labor, material, and regulatory costs are likely similar within the city (Glaeser and Gyourko, 2018).

We make use of our model, using the baseline elasticities and the current built space in NYC as shown in Table 1 to match two sets of moments. First, we use PLUTO information regarding the ratio of land value compared to the value of the property. We match this average to our model moment by setting the current built space to $F_i = 0$: $E[\pi(F_i = 0; \Theta)/\pi_i(F_i = F_i; \Theta) - 1] = 0.252614$.Second, the demolition cost shifter is explicitly matched using a piece of information from real estate developers who indicated to us that the value of the demolition cost usually accounts for 13-20% of the total development costs (i.e., demolition and construction costs). This calibration yields $k_c = 64.09$ and $k_d = 11.36$. To put this into perspective, note that the mean value of V_i in our model is 562.2 which implies that the cost of construction of a unit of density is around 11% the value of that density.

3.2 Global parameters calibration

Risk-free rate. To calibrate r, we use the average 3-month market yield on U.S. treasury securities from the Board of Governors of the Federal Reserve System (FRED). This series averages 3.79% over the available period (September 1981 to December 2023).

Property tax rate ψ. To compute the *effective* property tax rate on residential properties, we combine several datasets. First, we link property assessment data for multi-family properties from the Primary Land Use Tax Lot Output (PLUTO) with the statutory tax rates (NYC Department of Finance) to calculate property tax bills for all parcels in NYC since 2000. We then merge the tax bills with 36,000 transactions of multi-family properties from Costar. Lastly, we derive the effective tax rates by dividing the tax bills by the corresponding sale prices. Since 2000, this rate has averaged 1.34%, with an interquartile range of 0.56% to 1.8%.

3.3 Market-level parameters

We define market $j = \{1, 2, ..., J\}$ as the aggregation of residential parcels i located within a neighborhood, with n_j representing the total number of parcels. We chose community districts as our spatial unit of neighborhoods because zoning changes typically require approval at this local level. Each market is characterized by its return δ_j, return volatility σ_j, and the average market value per unit of density V_j.

Calibration of δ_j – total return to multi-family housing. We compute δ_j as the combined sum of property (asset) and rental (dividend) returns. Property returns are calculated at the borough level using Costar's multi-family asset transaction data from 2004 to 2023.[18] Property annual returns are observed to be highest in Manhattan (7.6%), followed by Brooklyn (6.8%), Queens (6.0%), and the Bronx (5.0%). Rental growth is then calculated at the neighborhood level using the average annual increase in median gross rent from 2013 to 2022, collected from the Census at the census block group level. For instance, the neighborhood encompassing Brooklyn's Williamsburg experienced an average annual rental growth of 7.4% over the past decade. Combining property and rental returns yields the distribution of δ_j, illustrated in Panel A of Figure 4, with an average return of 10.5%.

Calibration of σ_j using model's implied volatility. Conditional on the baseline elasticities and parameters, we determine the return volatility, σ_j, that best aligns our model's predictions with the observed housing production across markets. Specifically, we calibrate σ_j by matching the model's predicted number of housing units constructed to the actual housing units built in each neighborhood between 2010 and 2023, using data from NYC Planning. Specifically, calibrating the parcels' characteristics to 2010, we estimate σ_j by minimizing the difference between the model's predicted construction activity and the

[18]Due to the limited number of transactions per neighborhood, property returns are computed at the borough level. Staten Island's neighborhood returns are excluded from calibration for the same reason.

empirical data for each market j:

$$\arg\min_{\sigma_j} \left[\sum_{i \in j} \underbrace{\mathbf{E}_{2010} \left[\mathbf{P}_i(..., \sigma_j, \Theta | t = 13) F_i^*(..., \sigma_j, \Theta) \right]}_{\text{Model's expected units within 13 years given 2010 data}} - \underbrace{UnitsConstructed_j^{2010-2023}}_{\text{Data}} \right]. \quad (14)$$

We calibrate σ_j using the volatility implied by our model, drawing parallels to the concept of implied volatility introduced by Black and Scholes (1973) for the pricing of corporate securities (Dumas et al., 1998). The resulting average implied volatility is 16.1%, with variations across markets depicted in Panel B of Figure 4.

Calibration of V_j – mean value per unit of density. To calibrate V_j, we use the 2022 Census median gross rent, averaged across census block groups within each neighborhood. Applying a uniform capitalization rate across the city results in an average V_j of \$561,000, with variations illustrated in Panel C of Figure 4. Additionally, we use rent variations within each neighborhood to parameterize parcel-specific V_i (Panel D). The methodology for incorporating these variations into parcel-level parametrization is detailed below.

Affordable housing stochastic process assumptions. Unless otherwise specified, we calibrate the returns and volatility of affordable housing units using the market's stochastic processes. Affordable housing, often associated with social impact investment, generally yields lower returns. Thus, we set the return on affordable housing in market j to be 1% lower than the corresponding market-rate return, i.e., $\delta_{\alpha(j)} = \delta_j - 0.01$. However, given the reduced risk associated with high-demand affordable units, we assume that return volatility for affordable housing is 50% of that for market-rate units, such that $\sigma_{\alpha(j)} = \sigma_j/2$. Since both processes are linked to the same housing market, we assume a perfect positive correlation, setting $\sigma_{\alpha,j} = 1$.

3.4 Parcel-level parameters

Each parcel i is characterized not only by its location within market j but also by its current density F_i, maximum allowable density $\bar{F}_i$, operational obsolescence o_i, and a time-invariant quality factor q_i. The quality parameter q_i captures above- or below-average location attributes that, in turn, influence the parcel-specific market value per unit of density V_i. The range of these variables is summarized in Table 1. To maintain consistency with our analysis, we restrict the sample to parcels zoned or partially zoned for residential use, excluding those designated for public or other non-residential purposes.[19] In total, we use 672,241 parcels.

Built and maximum allowed density (F_i and $\bar{F}_i$). We calibrate F_i and $\bar{F}_i$ using the existing as-built Floor-Area-Ratio (FAR) and the maximum allowable FAR, because they represent key density measures. To ensure consistency, we normalize these values based on the observed relationship between FAR and the number of residential units, controlling for parcel size. This allows us to express F_i in terms of residential units per a standard parcel size of 2,500 square feet.[20] In our baseline calibration, F_i has an average value of 2.94, while $\bar{F}_i$ averages 4.11, with 35.2% of parcels built to their maximum allowable density.

Operational obsolescence o_i. We account for operational inefficiencies, such as energy loss, higher capital reserves, or vacancy, by allowing the current asset to yield a lower rental than the market it would absent this inefficiency. We capture this operational obsolescence using the functional form $o_i = 1-(1-\mu)^{RenovAge_i}$, where $RenovAge_i$ is the number of years since the last recorded renovation or construction and μ is the annual rental de-

[19] To keep parcels that can accommodate residential units, we, in effect, keep parcels that have an "R-x" zoning classification in either the first, second, third, or fourth zoning classification. In addition, we keep parcels with observed residential units regardless of their zoning classification.

[20] For instance, in the neighborhood encompassing Manhattan's Chelsea and Hudson Yards, a built FAR of one corresponds to 3.46 residential units on a typical 2,500-square-foot parcel, whereas in Queens, this statistic averages 2.53 units.

preciation rate. We us $\mu = 0.015$ based on the results of Lopez and Yoshida (2022) that estimate an annual rental depreciation for condominiums at 1.5% reflecting both structural and functional obsolescence. In our baseline calibration, operational obsolescence averages 63.0%.

Lot-specific quality shifter q_i. We account for within-market heterogeneity in values by allowing the market value per unit of density to vary across parcels. Factors such as proximity to metro stations, parks, or waterfront views contribute to these variations. To capture this heterogeneity, we define parcel-specific values as $V_i = V_j q_i$, where q_i is the time-invariant locational quality. We assume q_i is exogenous to other lot-level characteristics (e.g., built density, obsolescence) and draw it from a log-normal distribution with mean zero and standard deviation σ_{V_j}. The standard deviation is computed using the log residuals of median rents in market j, obtained from Census data.[21] This specification enables us to capture the quality variation present in neighborhoods with significant heterogeneity, such as Manhattan's Alphabet City, as illustrated in Panel D of Figure 4.

4 Baseline model outcomes

Before analyzing counterfactual outcomes, we first assess the accuracy of our baseline model, identify the key determinants of development, and evaluate the model's robustness to variations in underlying parameter assumptions.

4.1 Baseline outcomes

Using the parameterization described in the previous section, we compute key outcomes under our baseline scenario, where no policy interventions are applied ($\mathcal{P}_0 = \{\alpha = 0, \phi = 0, \tau = 0\}$). These include: (1) $\mathbf{P}_i(\mathcal{P}_0; \Theta|t)$ - the probability that a parcel under-

[21]Specifically, for each market j we retrieve the log residuals rents $\varepsilon_{b,j}$ of each block group $\log(Rent_{b,j}) = \log(V_j) + \varepsilon_b$, and compute the variance of $\varepsilon_{b,j}$ within j: σ_{V_j}.

goes redevelopment within a t-year horizon, (2) $\pi_i(\mathcal{P}_0;\Theta)$ - the parcel value accounting for both current use and its option value, and (3) $F_i^*(\mathcal{P}_0;\Theta)$ - the optimal redevelopment density. These baseline outcomes serve as our benchmark for evaluating counterfactual policy interventions.

Aggregate outcomes. We analyze several aggregate outcomes either at the city or market level, with a primary focus on the expected number of housing units constructed (or under construction) at the 10-year horizon, $\mathbf{E}[Apt|t=10]$. Additionally, we compute related measures, including the expected net addition of units ($\mathbf{E}[AptNet|t=10]$), expected density ($\mathbf{E}[Density|t=10]$), the present value of all parcels (Π), and the average value of the redevelopment option ($\mathbf{E}[Option|t=10]$). The market-level outcomes, for instance, are defined as:

$$\begin{aligned}
\mathbf{E}\left[Apt_j|\mathcal{P}_0,t=10\right] &= \sum_{i\in j}\mathbf{P}_i\left(\mathcal{P}_0;\Theta|t=10\right)F_i^*\left(\mathcal{P}_0;\Theta\right) \\
\mathbf{E}[AptNet_j|\mathcal{P}_0,t=10] &= \sum_{i\in j}\mathbf{P}_i(\mathcal{P}_0;\Theta|t=10)F_i^*(\mathcal{P}_0;\Theta)-F_i \\
\mathbf{E}[Density_j|\mathcal{P}_0,t=10] &= \frac{1}{n_j}\sum_{i\in j}\mathbf{P}_i(\mathcal{P}_0;\Theta|t=10)F_i^*(\mathcal{P}_0;\Theta)+\left(1-\mathbf{P}_i(\mathcal{P}_0;\Theta|t=10)\right)F_i \\
\mathbf{E}[\Pi_j|\mathcal{P}_0] &= \sum_{i\in j}\pi_i(\mathcal{P}_0;\Theta) \\
\mathbf{E}[Option_j|\mathcal{P}_0,t=10] &= \frac{\sum_{i\in j}\tilde{\pi}_i(\mathcal{P}_0;\Theta)}{\sum_{i\in j}\pi_i(\mathcal{P}_0;\Theta)}
\end{aligned} \tag{15}$$

where $\tilde{\pi}_i(\mathcal{P}_0;\Theta)$ is the value of the four last terms of equation (6).

We present the aggregated city-level outcomes in the first row of Table 3. The model estimates the expected number of housing units constructed at 211,996 units over a 10-year horizon.[22] This projection is consistent with historical construction trends in the city, which have ranged from 10,000 to 28,000 new units per year since 2010 (NYC housing pro-

[22] At this stage, our analysis does not incorporate crowd-out effects (Eriksen and Rosenthal, 2010) that could impact the total housing supply. Instead, we assume that real estate developers act as price-takers and are too small individually to influence citywide price levels.

duction). Accounting for the demolition of existing units, we estimate a net construction of 12,095 units per year over a 10-year period. Figure 5 illustrates the spatial distribution of new housing units (completed or under development) throughout the city in this baseline scenario.[23] The model predicts more development activity in areas of Brooklyn and Queens adjacent to the East River, as well as in Manhattan's Chelsea and Hudson Yards, reflecting the continued demand for housing in these prime locations.

At the 10-year horizon, the model projects an increase in average residential density from 2.88 to 3.11 units per 2,500 square feet of land. As expected, density declines progressively with distance from the center of Manhattan (Panel B of Figure 6), aligning with predictions from the monocentric city model (Alonso, 1964; Mills, 1967; Muth, 1969). Furthermore, our model estimates the total value of residential assets in New York City at approximately \$0.96 trillion. Our estimate is within the same order of magnitude as the valuation reported by the Department of Finance (Tax Lot Assessment Portal).

Finally, our model finds that the redevelopment option accounts for 2.00% of the total value of residential assets, a relatively low proportion that reflects that most of the parcels in the city are already developed. However, as shown in Panel D of Figure 6, some neighborhoods, particularly those with significant redevelopment potential, such as Long Island City in Queens, or those with aging building stock, such as parts of the Bronx, exhibit a higher share of option value, reaching up to 20%.

To further validate our model, we assess whether its predicted property values align with observed real estate prices in New York City. Specifically, we calculate the predicted price per unit for each parcel and average these values within each market, $1/n_j \sum_{i\in j}(\pi_i(\mathcal{P}_0;\Theta)/F_i)$. Figure 7 compares these model-implied prices with the unconditional average price per unit of multi-family transactions in each neighborhood, using Costar transaction data. The strong positive correlation ($\rho = 0.89$) between the

[23] Additional maps detailing other outcomes are provided in Figure 6 of the Appendix.

predictions of the model and the observed market prices underscores the accuracy of the model's calibration.

Parcel heterogeneity. To examine the role of parcel heterogeneity in shaping key real estate outcomes, we present in Table 4 an analysis of parcel-level characteristics and their influence on core model outputs, including the probability of development $\mathbf{P}_i$, property value $\log(\pi_i)$, and optimal redevelopment density F_i^*. We estimate partial correlations between these outcomes and parcel-level characteristics while controlling for neighborhood fixed effects.

Column (1) shows that the probability of development correlates positively with parcel quality and obsolescence, while the existing density acts as a deterrent. This result aligns with economic intuition: higher obsolescence lowers the value of current cash flows, making redevelopment more attractive, while taller buildings increase the opportunity cost of redevelopment. Column (2) shows that the optimal density of redevelopment is positively correlated with the quality of the plot, suggesting that developers tend to build taller buildings when potential cash flows justify the additional cost. However, the most influential predictor is the current allowable density, as developers tend to build to the maximum allowed density, a pattern documented in Büchler and Lutz (2024) and Brueckner et al. (2024).

Column (3) reveals that the value of the property is determined primarily by the quality of the parcel (positive), obsolescence (negative), and current density (positive). Even after controlling for building size (Column 5), taller buildings exhibit higher values, consistent with the findings on positive returns to height (Ahlfeldt and McMillen, 2018). Meanwhile, Column (4) highlights that the option value is negatively correlated with the existing as-built density and positively correlated with obsolescence. This finding reflects the tendency for redevelopment to occur on underutilized, aging properties. Overall, these findings emphasize the critical role of the existing built environment in shaping redevelopment activity and the importance of incorporating these factors into policy design.

Robustness to underlying parameters. Table 3 evaluates the sensitivity of aggregate outcomes to key elasticity assumptions, demonstrating how our model captures trade-offs between development costs and returns. Shorter construction durations increase expected apartment production, density, and property values, primarily by enhancing the value of the redevelopment options. In contrast, a greater height elasticity in construction costs leads to a significant reduction in new development and lower option values, highlighting the importance of construction technology in shaping urban development (Ahlfeldt et al., 2023).

The third row shows that when developers are constrained to build at the maximum allowable density, aggregate outcomes remain largely unchanged. In a city like NYC, where zoning regulations often constrain redevelopment (Brueckner et al., 2024), developers already build to the regulatory limit. Adjusting assumptions about the value per unit of density (V_i) reveals that increasing returns to height raises expected density and property values. However, unlike changes to construction costs, this effect does not translate into higher redevelopment option values.

5 Policy counterfactuals

This section quantifies the impact of policy instruments on urban development. To compute counterfactuals, we parameterize each city parcel as outlined in Section 3, using the parameter estimates in Table 1, unless otherwise stated. We evaluate policy interventions that modify the regulatory framework at time $t = 0$, moving from the policy environment $\mathcal{P}_0$ to $\mathcal{P} = \{\alpha \geq 0, \phi \geq 0, \tau \geq 0\}$. These changes introduce a combination of incentives (*Carrots*) and constraints (*Sticks*) that immediately shape developers' decisions regarding the optimal timing and density of development, ultimately affecting property values.

We denote counterfactual outcome deviations from the baseline outcomes with Δ. For instance, the market-level change in the expected number of units constructed under policy $\mathcal{P}$ is expressed as:

$$\begin{aligned}\Delta\mathbf{E}[Apt_j|\mathcal{P},t=10] &= \mathbf{E}[Apt_j|\mathcal{P},t=10] - \mathbf{E}[Apt_j|\mathcal{P}_0,t=10] \\ &= \textstyle\sum_{i\in j}\mathbf{P}_i(\mathcal{P};\Theta|t=10)F_i^*(\mathcal{P};\Theta) - \displaystyle\sum_{i\in j}\mathbf{P}_i(\mathcal{P}_0;\Theta|t=10)F_i^*(\mathcal{P}_0;\Theta)\end{aligned}$$

Thus, all policy counterfactual results are computed by measuring deviations from baseline outcomes, expressed as percentage changes for monetary outcomes. One additional aggregate outcome, not considered in the baseline case, is the number of expected affordable units, $\Delta\mathbf{E}[\text{Aff}_j|\mathcal{P},t=10] = \mathbf{E}[\text{Aff}_j|\mathcal{P},t=10]$ given that $\mathbf{E}[\text{Aff}_j|\mathcal{P}_0,t=10]=0$ since $\alpha=0$ in $\mathcal{P}_0$.

Beyond the expected number of total and affordable units constructed within a given time horizon, we also consider potential costs associated with policy implementation. Specifically, we evaluate changes in development density, the present value of tax revenues, and the total present value of NYC residential assets. This last metric is particularly relevant as it may introduce political risks—either from residents if values rise too sharply or from real estate industry stakeholders if values decline significantly. However, explicitly quantifying these monetary costs falls outside the scope of this study.

5.1 Policy counterfactuals benefits and costs

Figure 1 presents the core findings of our counterfactual policy analysis, illustrating the trade-offs between benefits and costs associated with various policy interventions. Panel A depicts the benefits, measured in terms of expected changes in affordable and market-rate housing production. Panel B shows two primary public costs: changes to overall density and changes in expected tax revenue.

The analysis considers six distinct policy interventions: (1) upzoning, which increases allowable density; (2) tax exemptions, which provide financial incentives for new development; (3) affordability mandates, which require a share of units to be affordable; (4) incentive zoning, combining affordability mandates with density bonuses; (5) fiscal incentives, pairing affordability mandates with tax exemptions; and (6) combined incentives, which incorporate all three mechanisms. A detailed description of these policies and their implications can be found in Sections 5.2 and 5.3.

5.1.1 Panel A: Public Benefits of Policy Interventions

We assume that an optimal policy to combat the affordability crisis seeks to enhance affordable supply while mitigating adverse impacts on overall market-rate housing production. This optimal policy is depicted by the -45 degrees red line whereby a shift as far to the right reflects an optimal expansion of affordability benefits. This approach inherently results in a net substitution effect, where market-rate units are replaced by affordable housing, striking a balance between density control and housing accessibility.

The results indicate substantial variation across policies. Upzoning (Policy [1]) generates the largest expected increase in market-rate housing but does not contribute to affordability. It results in the highest density increase, moving away from the affordability goal.Tax exemptions (Policy [2]) increase overall housing supply with minimal impact on density but do not contribute to affordability, as all new units built are market-rent. The Affordability Mandate (Policy [3]) directly replaces market-rate units with affordable housing, generating a strong substitution effect. Although it aligns with the affordability goal, it suppresses overall housing production, making it less effective in increasing total supply. Incentive Zoning (Policy [4]), which combines affordability mandates with density bonuses, produces affordable units while keeping overall supply unchanged. Fiscal Incentives (Policy [5]), which pair affordability mandates with tax exemptions, increase affordable units at the cost of reducing overall housing production. Finally, the Combined Incentives policy (Policy [6]), which incorporates affordability mandates, density bonuses, and tax exemptions, achieves the highest number of affordable units but increases overall supply over the baseline case.

5.1.2 Panel B: Costs of Policy Interventions

Panel B captures the financial and urban planning trade-offs associated with each policy. The two primary cost dimensions include (i) the fiscal burden, measured by changes in expected tax revenue (y-axis), and (ii) the increase in overall density (x-axis)[24]. A cost-neutral policy (relative to the baseline) is positioned as close to the origin as possible, minimizing both fiscal losses and density increases.

The results show that policies impose varying costs on local governments and urban infrastructure. Upzoning (Policy [1]) generates a substantial increase in density but has a negligible effect on tax revenues. Although it successfully expands housing supply, the strain on infrastructure and public services intensifies due to higher population density. Tax Exemptions (Policy [2]) substantially reduce tax revenues while causing a moderate rise in density, prioritizing housing supply expansion at the expense of fiscal sustainability. The Affordability Mandate (Policy [3]) leads to a modest decline in tax revenue but reduces expected density, as affordability requirements disincentivize new development. Incentive Zoning (Policy [4]) results in notable density increases while keeping tax revenue losses relatively controlled, striking a balance between supply growth and affordability mandates. Fiscal Incentives (Policy [5]) cause the most significant reduction in

[24]Higher density imposes costs on public infrastructure and essential services, increasing demand for roads, transit, utilities, schools, and emergency response. It also contributes to congestion, environmental strain, and higher maintenance expenses. These factors must be weighed when assessing the net impact of density-enhancing policies.

tax revenues yet have a minimal effect on density, making affordability gains highly dependent on sustained public sector investment. The Combined Incentives policy (Policy 6) presents the highest costs across both dimensions, substantially diminishing tax revenues and significantly increasing density. Although this policy achieves the strongest affordability outcomes, it places the greatest financial burden on the public sector.

The results highlight the trade-offs policymakers must navigate when designing zoning and fiscal incentives for stimulating housing construction. Upzoning (Policy [1]) expands market-rate housing supply with minimal fiscal cost but does not guarantee affordability. In contrast, fiscal and density incentives (Policies [4] and [5]) support affordability objectives but impose significant costs in terms of forgone tax revenue or increased urban density. A balanced approach may be found in combined incentive zoning (Policy [6]), which integrates multiple policy tools to expand affordability at a higher cost on both margins. Ultimately, the choice of policy depends on how jurisdictions prioritize affordability, fiscal sustainability, and urban planning considerations.

5.2 Unilateral policies

Unilateral zoning and fiscal interventions affect affordable housing production, market-rate supply, total net new supply, density growth, and tax revenue losses in distinct ways. Table 5 evaluates three key standalone policies: Upzoning (Density Bonus of 25%), Tax Abatement (30-year exemption), and an Affordability Mandate (20% affordable). The objective is to assess how each policy influences housing affordability while minimizing density increases and fiscal strain.

Upzoning increases total housing supply more than any other policy, adding 64,589 net new units, but does not create dedicated affordable housing. In contrast, an Affordability Mandate generates 32,613 affordable units by shifting production away from market-rate supply, which declines by 54,831 units. Tax Abatements incentivize development but with more limited supply effects, adding 19,821 net new units while failing to ensure affordability gains.

From a cost perspective, Upzoning significantly increases density (+3.09%) while preserving tax revenues, requiring infrastructure expansion. The Affordability Mandate minimizes density growth (-1.06%) but does not expand total supply, instead substituting market-rate units with affordable ones. Tax Abatements, while modest in density impact (+0.95%), carry the largest fiscal cost, reducing tax revenue by 1.16%.

In sum, Upzoning maximizes total supply and maintains municipal revenues but does not address affordability. The Affordability Mandate redirects market-rate development toward affordable housing but sharply reduces overall market-rate supply. Tax Abatements encourage moderate supply growth but impose the highest fiscal burden. These trade-offs, summarized in Table 5, highlight the limitations of standalone policies in achieving affordability without additional incentives.

5.3 Mandatory Carrot & Stick Zoning

Policies that combine affordability mandates with zoning and fiscal incentives yield different trade-offs in affordable housing production, market-rate supply, total net new supply, density growth, and tax revenue losses. Table 5 compares three key interventions: Density Incentive Zoning (affordability mandate + density bonus), Fiscal Incentive Zoning (affordability mandate + tax exemption), and the Combined Incentive Policy (density bonus + tax exemption + affordability mandate). These mixed approaches referred to as *Carrot & Stick Zoning*, attempt to mitigate the trade-offs seen in standalone policies. The goal is to maximize affordability gains while minimizing new supply growth, density increases, and fiscal losses.

The highest affordability gains come from the Combined Incentive Policy, which generates 54,558 affordable units, outperforming Density Incentive Zoning (45,633) and Fiscal

Incentive Zoning (38,743). However, Density Incentive Zoning achieves affordability by shifting market-rate production, reducing it by 19,216 units while adding 26,417 net new units. In contrast, Fiscal Incentive Zoning limits total new supply, producing 4,767 fewer net units than the baseline, while imposing a 0.90% tax revenue loss. The Combined Incentive Policy minimizes the decline in market-rate housing to just 157 units but results in the largest fiscal and density costs, with a 1.11% tax loss and a 2.60% density increase.

In sum, Fiscal Incentive Zoning is the most effective at limiting total new supply while increasing affordability but is costly to city finances. Density Incentive Zoning preserves tax revenue but results in a larger increase in net new units. The Combined Incentive Policy generates the highest number of affordable units and prevents a decline in total supply but significantly increases density and imposes the largest fiscal burden. These trade-offs, detailed in Table 5, underscore the need to balance affordability goals with fiscal and urban planning constraints.

5.4 Voluntary Carrot & Stick Zoning: Evaluating Policy Adoption and Market Responses

Allowing developers to voluntarily opt into affordability mandates paired with density and fiscal incentives results in nearly the same affordable housing production as mandatory adoption but at significantly lower fiscal and infrastructure costs. Table 5 includes voluntary Density Incentive Zoning, Fiscal Incentive Zoning, and the Combined Incentive Policy, showing that flexibility in adoption creates a market-aligned affordability strategy, leveraging private capital to meet public housing goals while reducing financial strain on the city.

Under voluntary adoption, developers choose to opt into a new policy regime $\mathcal{P}$ only if it provides higher property value than the status quo, i.e., if $\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$. This decision rule ensures that only parcels where incentives outweigh the regulatory burden participate, effectively self-targeting affordability incentives to the most economically viable projects. As a result, voluntary policies avoid the inefficiencies of blanket mandates by aligning developer incentives with affordability goals.

Voluntary adoption maintains high affordability outcomes. The voluntary Combined Incentive Policy produces 52,015 affordable units, nearly matching the 54,558 affordable units achieved under the mandatory policy in Section 5.3. Similarly, voluntary Density Incentive Zoning and Fiscal Incentive Zoning generate 43,384 and 42,715 affordable units, respectively, only slightly off from their mandatory counterparts (45,633 and 38,743). This suggests that developers are still strongly incentivized to participate even without compulsion.

The key advantage of voluntary adoption lies in dramatically reduced costs. Tax revenue losses drop significantly across all policies, with voluntary Density Incentive Zoning keeping tax revenue unchanged and voluntary Fiscal Incentive Zoning reducing tax losses to just 0.14%, compared to 0.90% under the mandatory version. Likewise, the voluntary Combined Incentive Policy cuts tax losses, from 1.11% to 0.97%. These reductions reflect targeted participation by developers who benefit most from incentives, allowing the city to extend affordability programs without a blanket fiscal burden.

Density increases also decline sharply. Under voluntary adoption, the Combined Incentive Policy leads to a density increase of just 1.98%, compared to 2.60% under mandatory participation. Similarly, voluntary Density Incentive Zoning results in only a 0.23% density increase, far lower than the 1.26% observed in the mandatory case. This moderation in density growth alleviates pressure on infrastructure, making voluntary adoption a more sustainable affordability strategy.

The effectiveness of voluntary adoption stems from developer selectivity. While only 14.0% of parcels opt into voluntary Density Incentive Zoning or Fiscal Incentive Zoning, adoption jumps to 38.6% when both density and fiscal incentives are bundled. This selective participation helps concentrate affordable housing production in areas where private development interest aligns with policy goals, reducing the need for costly citywide mandates.

In sum, voluntary Carrot & Stick Zoning delivers nearly the same affordable housing gains as mandatory adoption but at a fraction of the fiscal and infrastructure costs. This result highlights the potential for market-driven public-private collaboration, where policy flexibility encourages private developers to meet public affordability goals with minimal strain on city budgets and infrastructure.

5.5 Limitations of the counterfactual analysis

Although our counterfactual analysis provides valuable insights into the potential impacts of zoning and fiscal policy changes, it is important to acknowledge its limitations.

A key assumption underlying our model is the exogeneity of underlying asset values. However, in scenarios where housing supply increases substantially—such as under large-scale upzoning—the assumption of exogenous asset values may not hold. Specifically, our model does not explicitly incorporate general equilibrium effects that could arise from a significant expansion of housing supply. In a market such as New York City, where aggregate housing supply constraints have historically driven high property values, an increase of 86,000 new units—representing a 3.9% increase over baseline levels—may not substantially impact citywide rental prices. However, if the additional supply is highly localized, price adjustments could be more pronounced, potentially altering the incentives for development (Asquith et al., 2023).

Another potential limitation is that our model does not account for behavioral responses from developers beyond the decision to redevelop. In reality, developers may strategically time their investments based on anticipated regulatory changes or adjust the quality of construction in response to policy incentives. Future iterations of this analysis could incorporate a broader set of responses, including strategic delays in development, negotiations with the public sector, or substitution between different asset classes (e.g., office-to-residential conversions).

Finally, our counterfactuals focus on broad citywide policy interventions, but a more refined analysis could assess the effects of targeted policies applied to specific submarkets or parcels with high redevelopment potential. In future research, we will extend our framework to analyze more spatially targeted policies, which may provide additional insights into the heterogeneous effects of zoning reforms across different neighborhoods.

6 Conclusion

We present a general framework based on a real options approach to analyze the timing and intensity of capital replacement. The focus of our framework is on the role of policy instruments in shaping these decisions. We apply this framework to the redevelopment of existing buildings in urban areas, examining the impact of density bonuses, tax exemptions, and public good provision mandates on redevelopment decisions, investment intensity, and land values. Although our model is tailored to real estate redevelopment, it also applies to other durable capital investment decisions.

Using the NYC affordable housing mandate as an application, we calibrate our model to incorporate key aspects of urban development. These factors include the cost and return to height, uncertainty in future market conditions, and spatial heterogeneity in land values and returns. The results of our baseline model are consistent with the observed development patterns in the city. This highlights our model's robustness in predicting key outcomes such as housing supply, density changes, and redevelopment probability.

We next perform counterfactual policy analyses in which we document the balance between producing substantial affordable housing and maintaining positive net additions to market-rate units. Our analysis highlights the importance of considering the existing built environment and heterogeneity across parcels when designing public policies. For example, parcels that have either a high current density or a low redevelopment potential respond differently to policy interventions compared to parcels associated with a low density or significant redevelopment options. The insight resulting from this spatial variability is that targeted zoning reforms or incentive programs should endeavor to maximize their effectiveness while minimizing unintended consequences. Furthermore, while we quantify changes in aggregate outcomes such as housing supply, property values, and tax revenues, we do not assign specific monetary values to costs associated with density increases, political risks, or infrastructure needs. Although we recognize that these externalities represent critical factors for policymakers to consider when implementing these strategies, they are beyond the scope of this study.

Despite the focus on the affordable housing mandate, our model is flexible in accounting for other policy considerations, including but not limited to changes in land use (e.g., office-to-residential conversions) and incentive zoning for public plazas and infrastructure. Future research could extend our framework to incorporate these additional considerations, especially given the rising importance of adaptive reuse in urban redevelopment.

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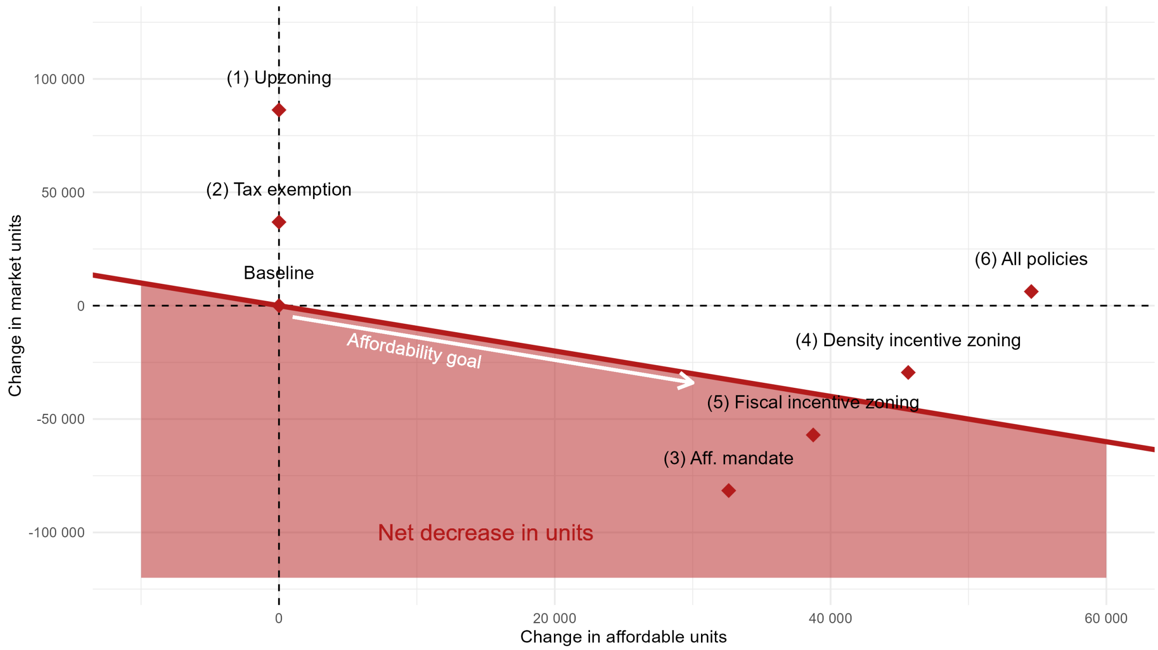
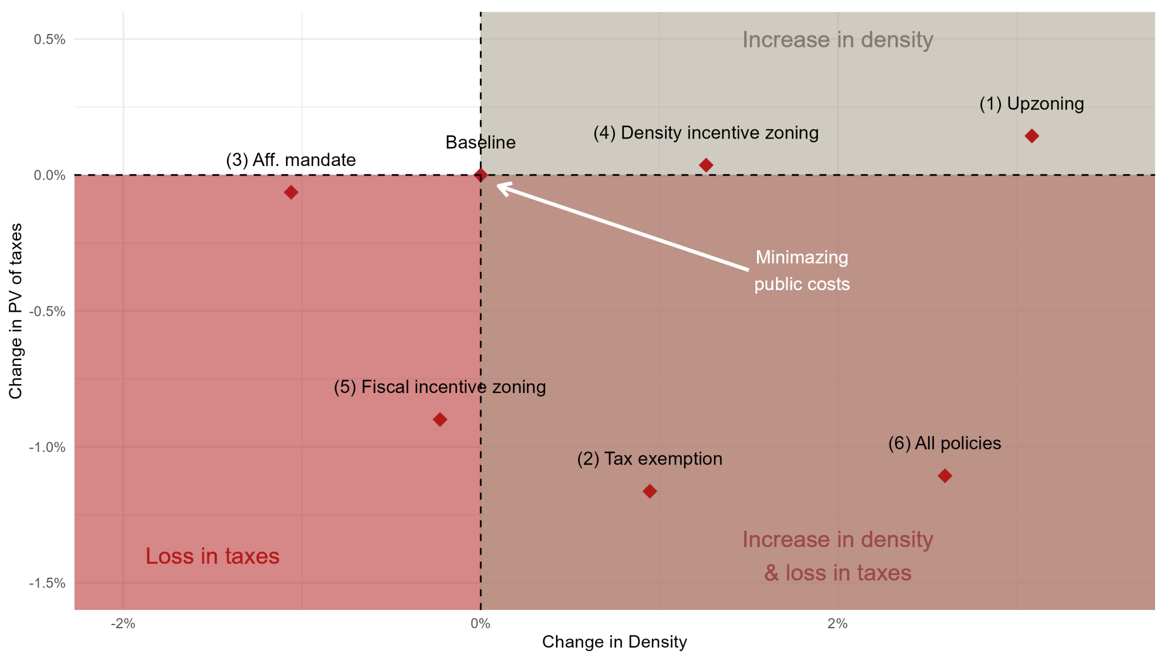
Figure 1: Policy counterfactuals benefits and costs



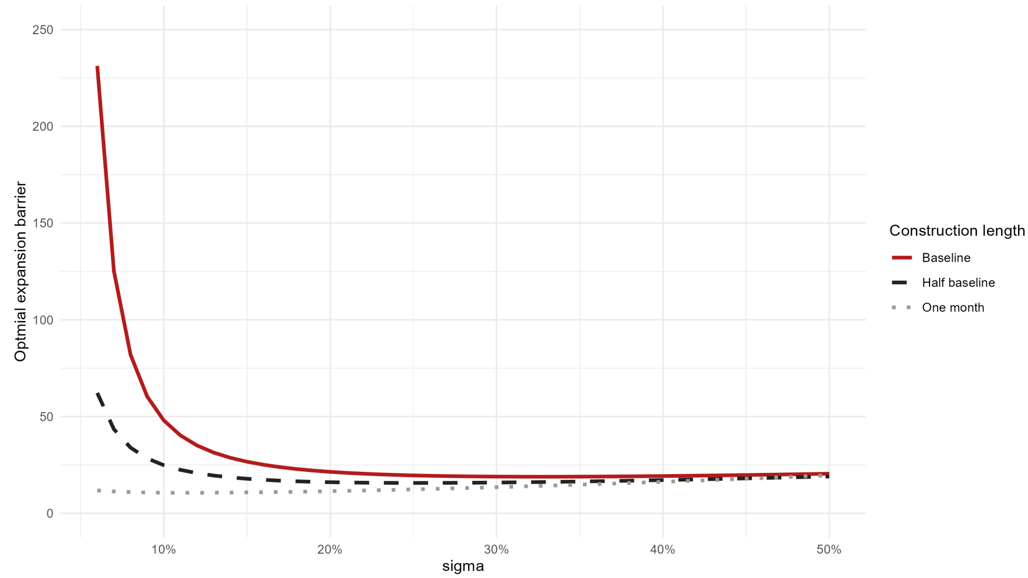
A. Benefits – Housing production



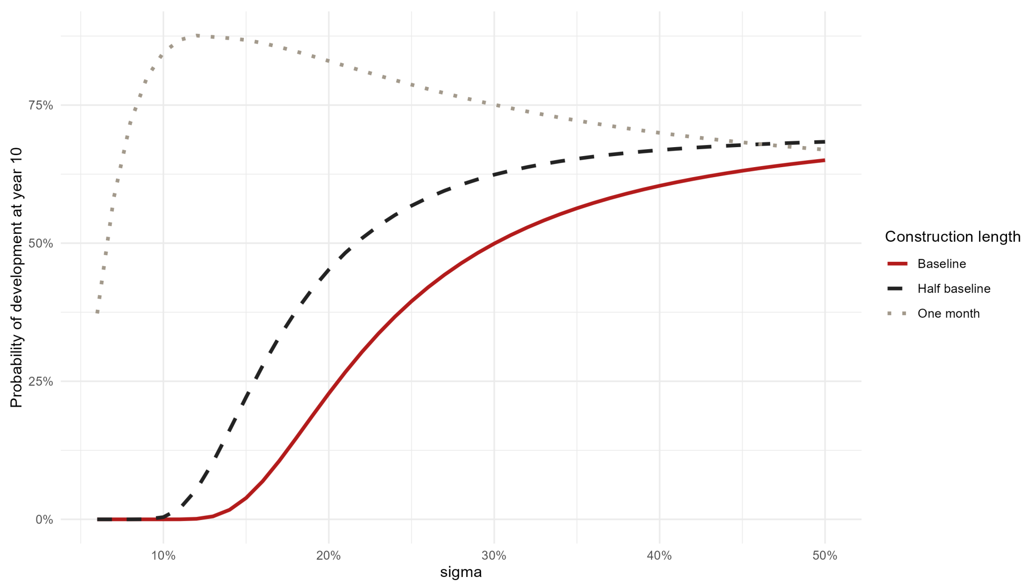
B. Costs – Public expenses

Note: This chart summarizes the policy counterfactual results. Panel A shows the expected change in affordable (x-axis) and market-rent units (y-axis). Panel B shows the expected additional costs for the city, including the change in density (x-axis) and the change in expected tax revenue (y-axis). Policy (1) Upzoning corresponds to an increase in allowable density of 25%, Polity (2) Tax exemption corresponds to a 30-year property tax exemption, and Policy (3) corresponds to a 20% affordability mandate. Policies (4), (5), and (6) are combinations of these. All of these counterfactuals are computed above the baseline scenario over a 10-year horizon.

Figure 2: Uncertainty, construction length, and probability of development



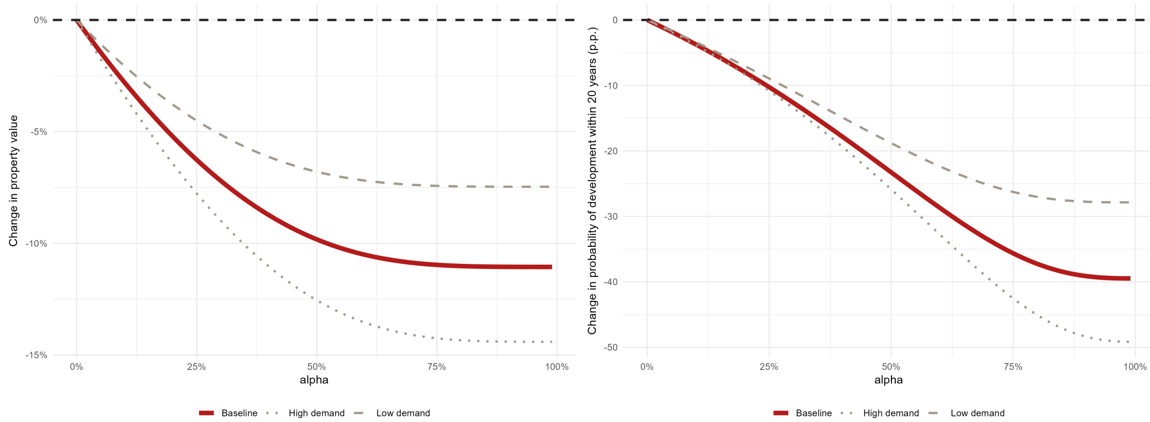
A. Optimal expansion barrier - V^{e*}



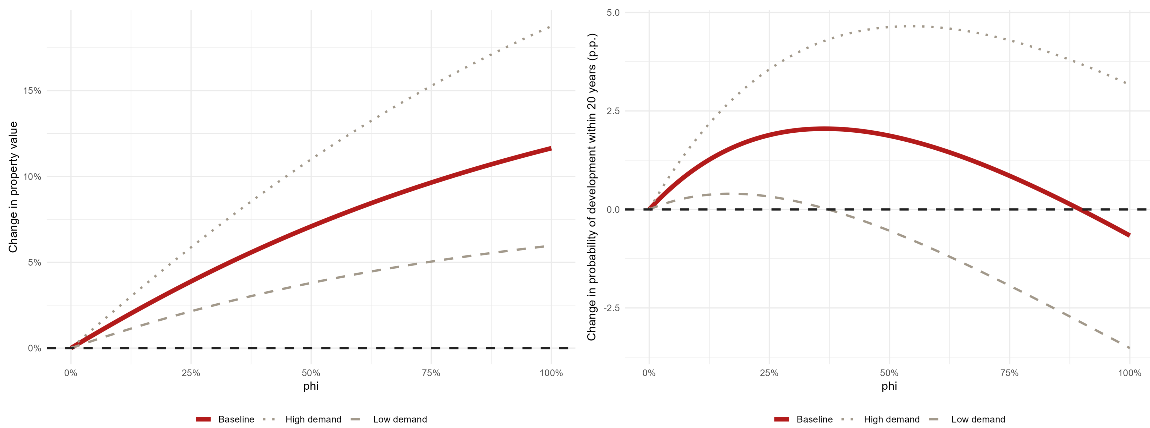
B. Probability of development - $\mathbf{P}\left(T^{V^{e*}} \leq 10\right)$

Note: This chart shows the effects of varying the underlying volatility σ_i on the optimal expansion barrier (panel A) and the probability of development within 10 years (panel B). Each line shows this effect for different assumptions regarding the length of construction (baseline: $d = 2.64$ years. These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 1.

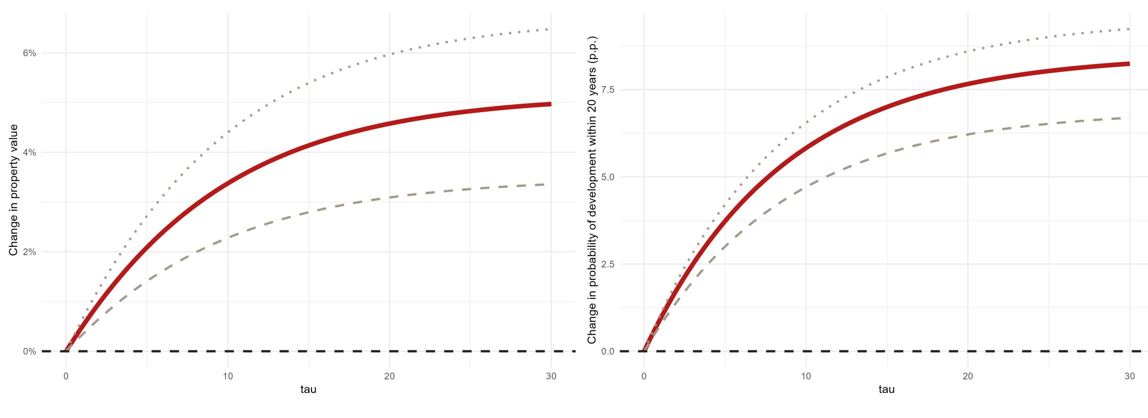
Figure 3: Property Value and Probability of Development with respect to Policy Instruments



A. Mandatory share for public goods (α)



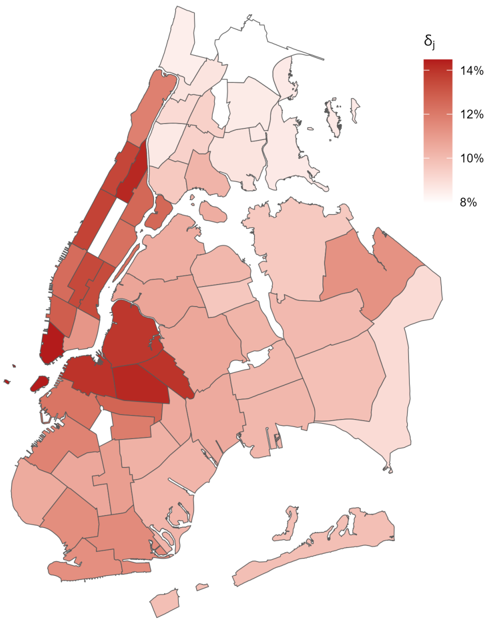
B. Density Bonuses (ϕ)



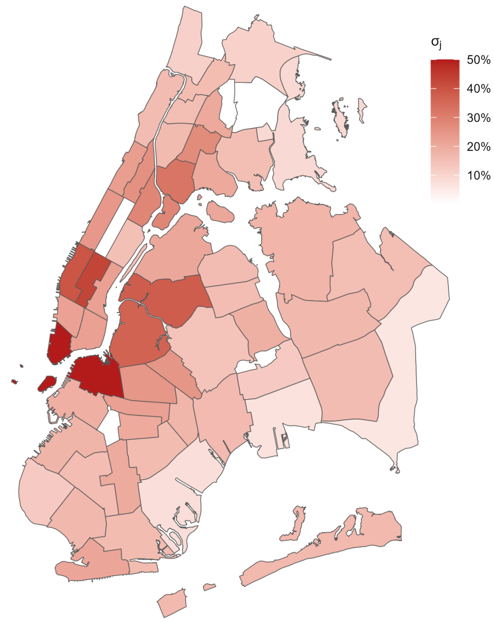
C. Length of Tax Exemptions (τ)

Note: These figures show the effects of changing the policy instruments on the change in the property value (left panels) and the change in the probability of development starting within 10 years (right panels). These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 1.

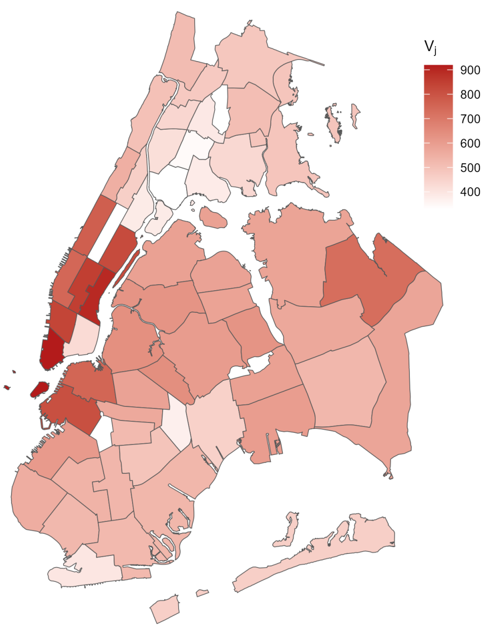
Figure 4: Market-level parameters calibration



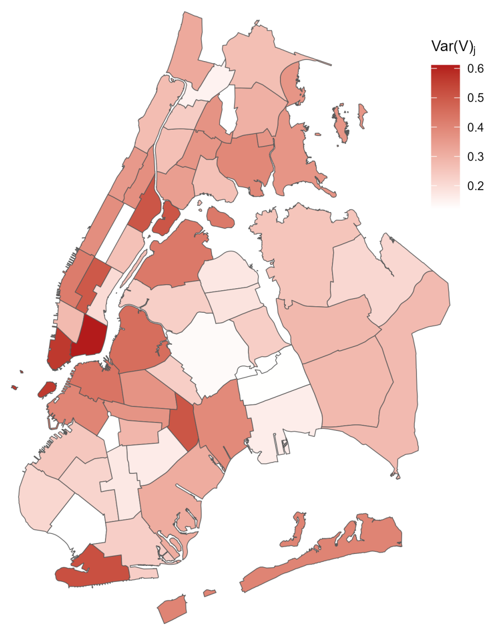
A. Return (δ_j)



B. Return volatility (σ_j)



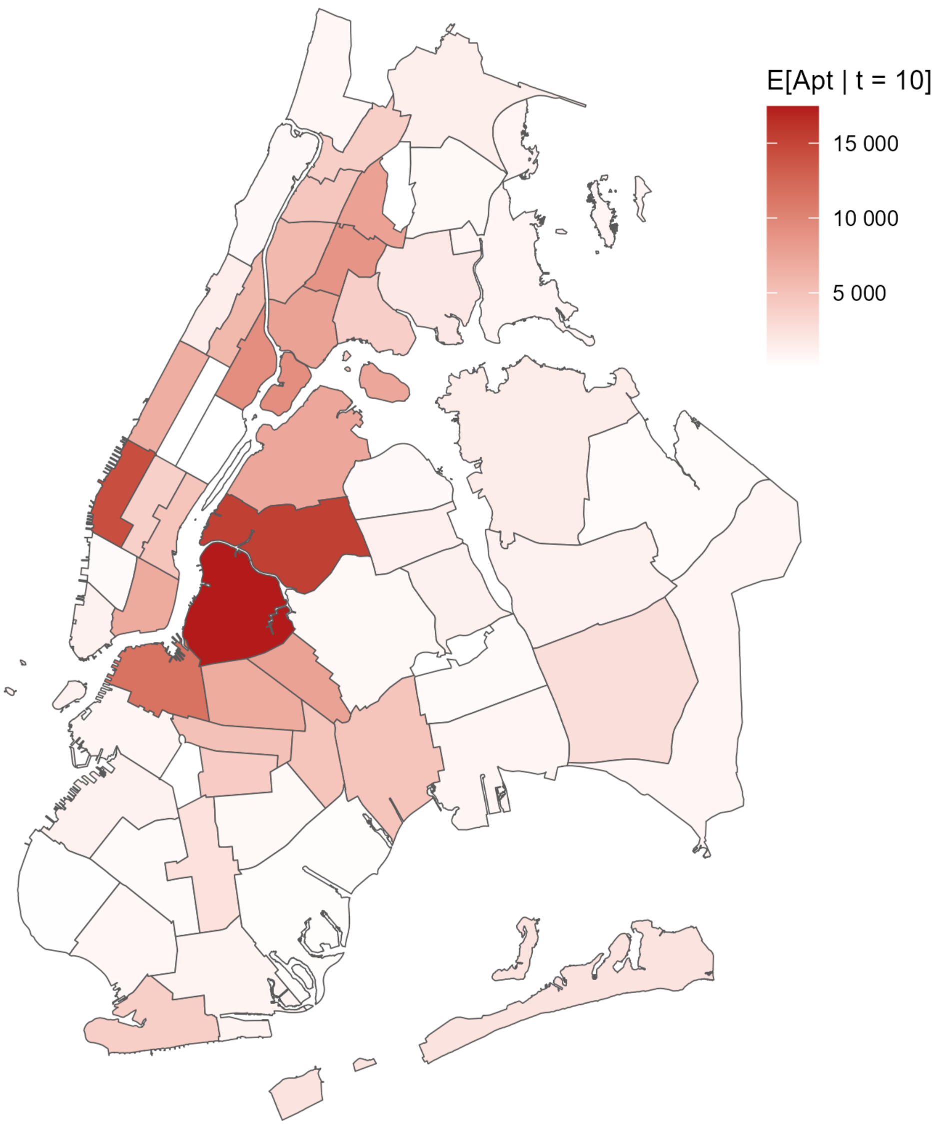
C. Market-value per unit of density (V_j)



D. Std. dev. of the log residuals of V_j (σ_{V_j})

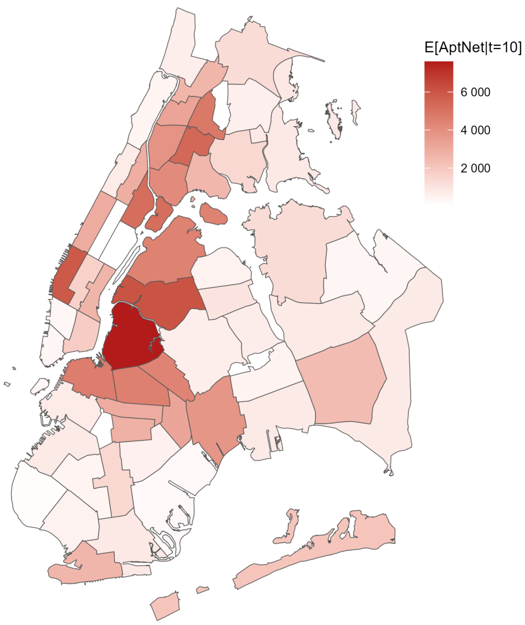
Note: These maps show the market-level parameters used in the baseline calibration and counterfactual analyses. In Panel A, we show the return on equity (the sum of property and rental returns) of multi-family residential assets. In Panel B, we show the return volatility implied by our model using targeted market-level construction intensity. In Panel C, we show the 2022 calibration of the market-value per unit of density V_j *(in \$000's). In Panel D, we show the standard deviation of the log residuals of median rent.*

Figure 5: Baseline calibration – Expected number of apartments

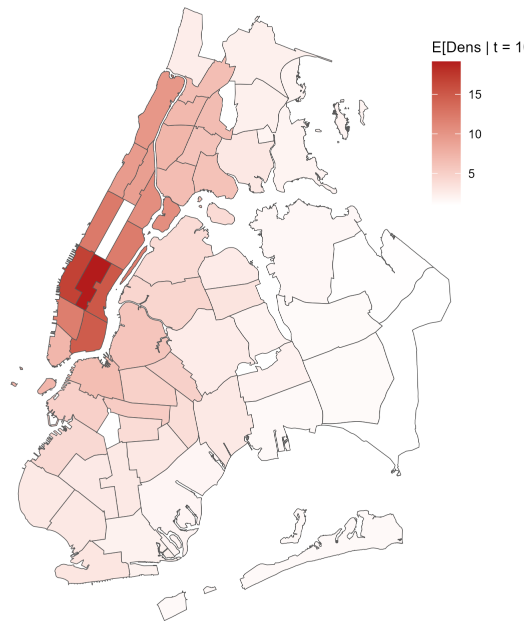


Note: This map shows the expected number of apartments constructed in each community district within a 10-year horizon, $E[Apt_j|\mathcal{P}_0, t = 10] = \sum_{i \in j} \mathbf{P}_i^{dev}(\mathcal{P}_0, \Theta|t = 10) F_i^*(\mathcal{P}_0, \Theta)$ using the model baseline calibration with parameters described in Table 1.

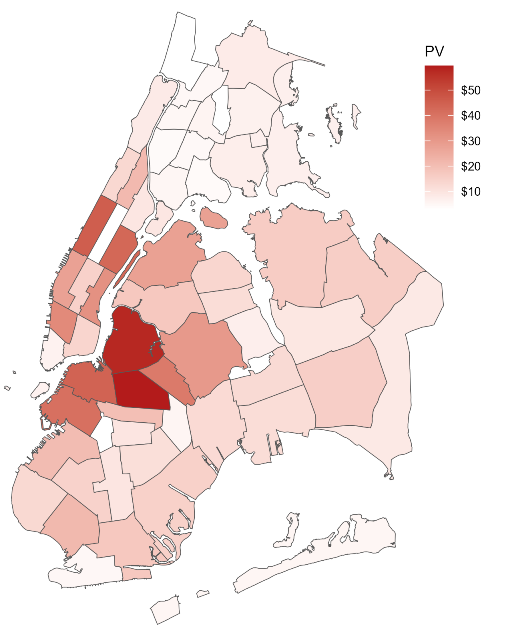
Figure 6: Baseline - Market-level aggregated outcomes



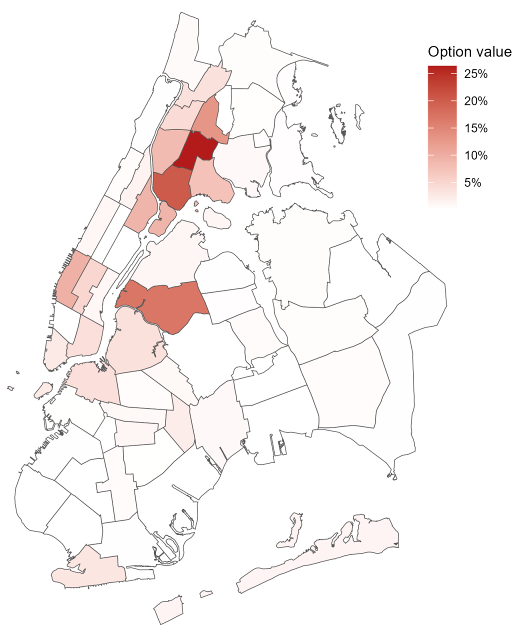
A. Expected net units



B. Expected density



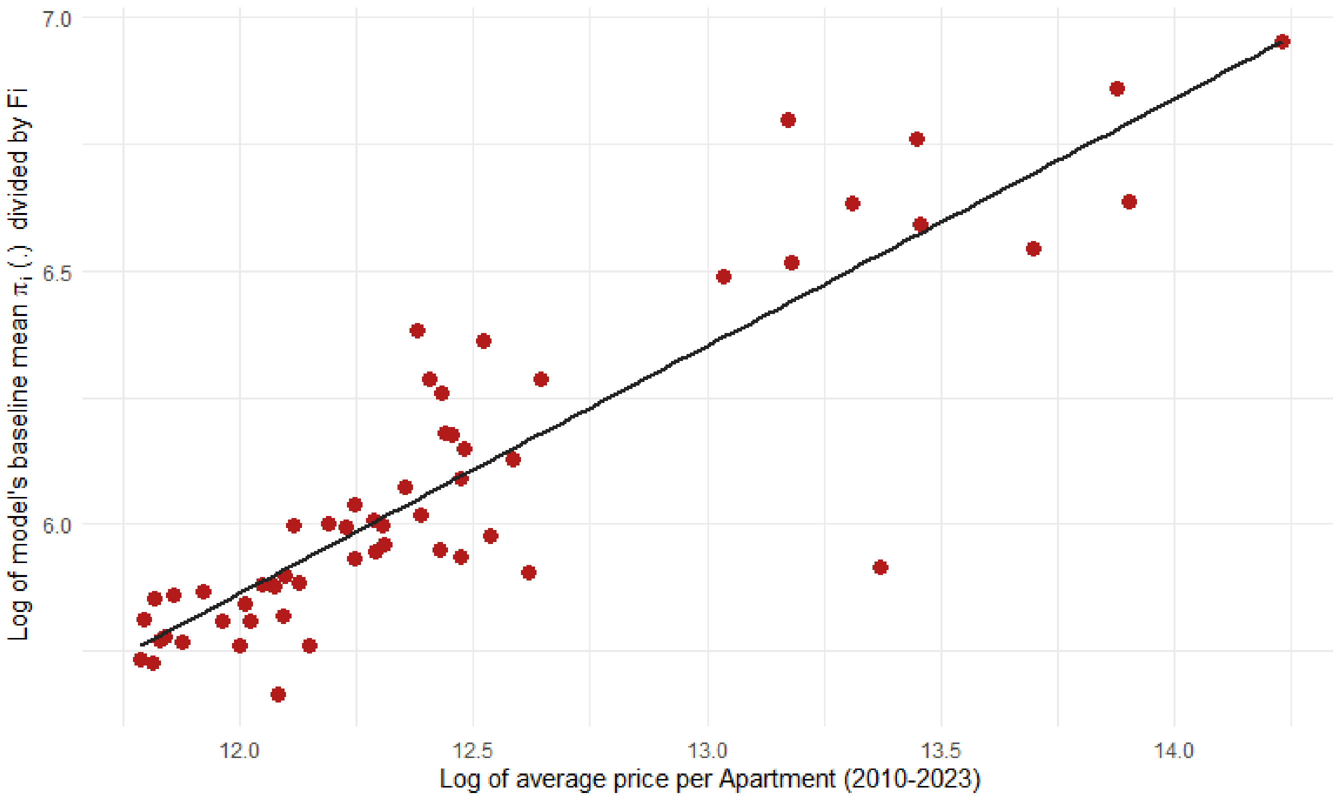
C. Present value of residential parcels



D. Share of redevelopment option value

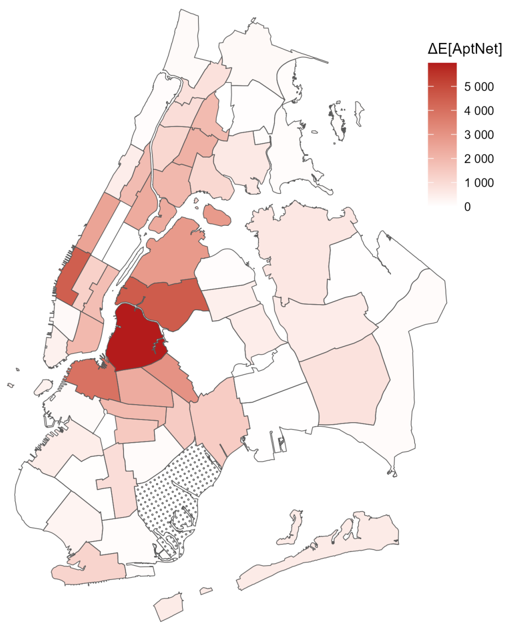
Note: These maps show the market-level aggregated outcomes of our baseline simulation as described in Table 1. Panel A showsthe expected net units addition, Panel B shows the expected density in 10 years, Panel C shows the sum of the residential parcel present values, and panel D the average value of the redevelopment option.

Figure 7: Model Fit

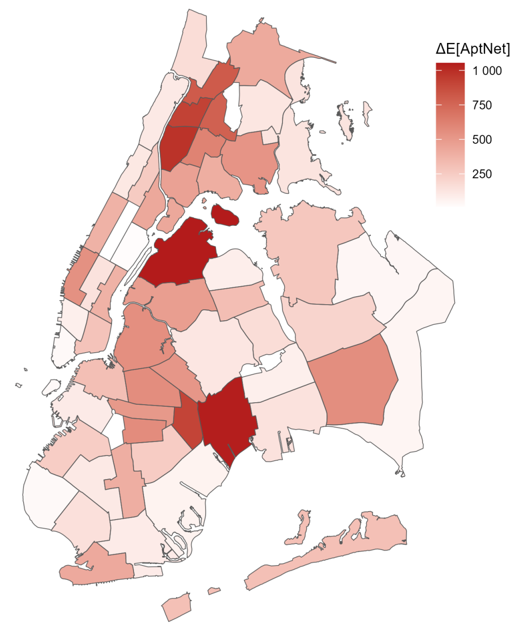


Note: This scatter plot shows the relationship between the log of the model's mean price per unit of density $\sum \pi_i / F_i(.)$ *in each NYC community district against the log of the unconditional mean price per apartments of Multi-Family buildings using transaction from CoStar since 2010. .*

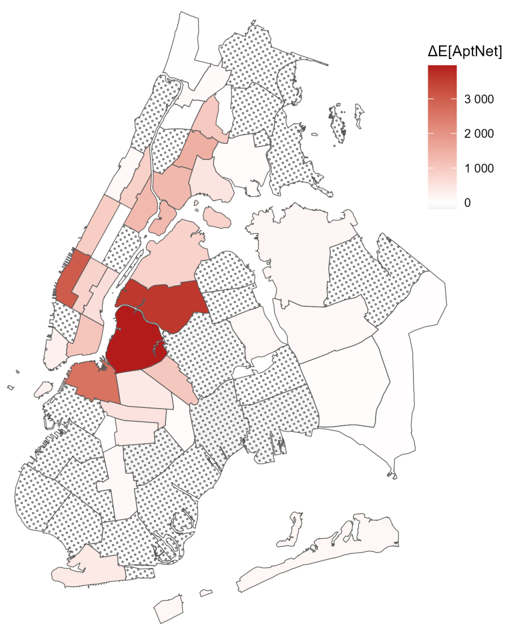
Figure 8: Counterfactuals – Net change in housing units



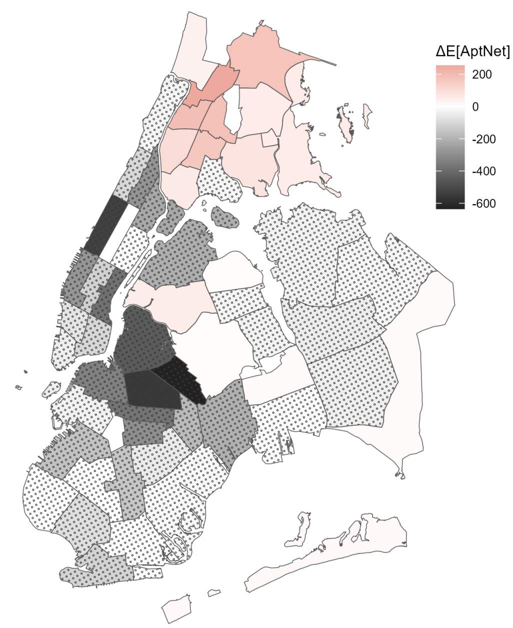
A. Upzoning ($\phi = .25$)



B. Tax exemptions ($\tau = 30$)



C. Density incentive zoning ($\alpha = 0.2$ & $\phi = .25$)



D. Fiscal incentive zoning ($\alpha = 0.2$ & $\tau = 30$)

Note: These maps show the expected net change in the number of housing units constructed for different counterfactual policy analyses. It shows for each market the change compared to the baseline outcomes with horizon 10-year as shown in Figure 5.

Table 1: Baseline model elasticities and parameters

This table reports the baseline parameters of the vector Θ. The sources, calibrations, and estimations are detailed in Sections 3.

Parameters	Description	Value	Calibrated method & main source
	The Price function		
ϵ_F	Density elasticity of value	0.07	Literature
ϵ_α	Non-market units share elasticity of price	0.17	Estimation – Table A1
	The demolition cost function		
η_D	Demolition cost elasticity of density	0.72	Target: Fixed cost demolition (55%)
k_d	Baseline demolition cost per FAR	11.36	Target: Share of demolition cost (15%)
	The construction cost function		
η_F	Density elasticity of construction cost	0.60	Literature
η_α	α elasticity of construction cost	0.20	Estimation - RCLO report
k_c	Baseline construction cost per FAR	64.09	Target: Land-value ratio (25.26%)
	Others		
ψ	Property tax rate (%)	1.08	Estimation - DoF & Costar
d	Construction periods (years)	2.64	Estimation – DCP
r	Risk free rate (%)	3.79	Estimation – FED
	Market-level parameters		
V_j	Market-value per unit of density (000's)	[330.30-918.80]	Estimation – Census Rent
δ_j	Drift rate of stochastic process	[0.080-0.14]	Estimation – Census Rent
σ_j	Volatility of stochastic process	[0.091-0.50]	Target: observed housing production
$\delta_{\alpha(j)}$	Affordable housing return	[0.07-0.13]	Estimation – Census Rent
$\sigma_{\alpha(j)}$	Affordable housing return volatility	[0.046-0.25]	Target: observed housing production
	Parcel-level parameters		
F_i	Built density (in apartments per 2,500 sqft of parcel)	[0.00-70.22]	PLUTO (standardized)
$\bar{F}_i$	Maximum density (in apartments per 2,500 sqft of parcel)	[1.00-70.22]	PLUTO (standardized)
o_i	Obsolescence cost	[0.00-0.95]	Estimation – PLUTO (building age)
q_i	Parcel time-invariant quality	[0.13-6.90]	j specific random draw – Census Rent

Table 2: Parcel level comparative statics and sensitivity analyses

This table reports the change in property value and the change in the probability of development within 10 years with respect to the policy instruments. These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 1.

	Change in property value (%)	Change in prob. dev. (p.p.)
Mandating 20% of affordable housing ($\alpha = 0.20$)		
Baseline	-5.22	-7.97
Higher demand ($V = 2V$)	-8.96	-8.23
Higher affordable rental discount ($\epsilon_\alpha = 2\epsilon_\alpha$)	-6.11	-10.87
Higher affordable construction discount ($\eta_\alpha = 2\eta_\alpha$)	-4.84	-6.42
Increase maximum allowed density by 50% ($\phi = 0.50$)		
Baseline	7.09	1.87
Higher demand ($V = 2V$)	24.14	12.06
Higher value from height ($\epsilon_F = 1.5\epsilon_F$)	9.65	3.34
Higher cost from height ($\eta_F = 1.5\eta_F$)	0.34	-4.04
20-years property tax exemption ($\tau = 20$)		
Baseline	4.58	7.66
Higher demand ($V = 2V$)	9.31	9.88
Higher property tax rate ($\psi = 1.5\psi$)	8.37	12.13

Table 3: Baseline calibration outcomes and robustness to parameter change

This table reports the aggregate outcomes of our baseline calibration and additional sensitivity analyses performed by varying underlying model's parameter assumptions. The first column shows the 10-year horizon expected number of units constructed (or under construction), the second shows expected net addition, the third the expected density in 10 years, the fourth the sum of the residential parcel present values, and the fifth the average value of the redevelopment option. The first line shows the results using the baseline calibration described in Table 1, and the other lines show the same outcomes by varying one of the underlying model assumptions or parameters as described.

	$E[Apt\|t=10]$ # of Apt.	$E[AptNet\|t=10]$ # of Apt	Avg. Density per parcel	PV(taxes) ($billion)	PV ($billion)	Option Value (%)
	(1)	(2)	(3)	(4)	(5)	(6)
Baseline	$211,996$	$120,955$	3.114	271.83	959.38	2.00
	Varying underlying model's assumptions					
Shorter duration time ($d = d/2$)	$362,793$	$189,266$	3.216	272.55	971.08	4.05
Higher cost from height ($\eta_F = 1.5\eta_F$)	$131,471$	$70,854$	3.039	271.77	956.76	1.31
Higher value from height ($\epsilon_F = 2\epsilon_F$)	$246,017$	$143,445$	3.147	305.53	$1,088.45$	2.34
No endogenous density choice	$217,529$	$126,494$	3.122	271.83	959.27	1.98

Table 4: Partial correlations between inputs and outcomes of baseline model

*This table reports the partial correlations between key inputs (rows) and outcomes (columns) of our baseline model results. The partial correlations are conditional on community districts as well as all other inputs listed in the rows. It uses the outcomes of the model with horizon 10-years for the 672,241 city under the baseline calibration described in Table 1. In Column (1), the outcome is the estimated probability of redevelopment decision $\mathbf{P}_i^{\mathbb{P}}(\mathcal{P}_0;\Theta)$. In Column (2), the present value of the parcel $\pi_i(\mathcal{P}_0;\Theta)$. In Column (3), the option value share of total value $\tilde{\pi}_i(\mathcal{P}_0;\Theta)/\pi_i(\mathcal{P}_0;\Theta)$. In Column (4), the log of the total value per unit of built units $\pi_i(\mathcal{P}_0;\Theta)/F_i$ computed for parcels with positive construction. Pearsons standard errors around the correlation estimates are presented in parentheses. Estimates followed by ***, **, and * are statistically significant at the 1%, 5%, and 10% levels, respectively.*

	Baseline outcome				
	Prob	F opt	log(PV)	Option	log(PV/F)
	All parcels				*Built parcels*
	(1)	(2)	(3)	(4)	(5)
Time-invariant quality (q_i)	0.179^{***}	0.043^{***}	0.556^{***}	0.106^{***}	0.686^{***}
	(0.005)	(0.005)	(0.003)	(0.005)	(0.003)
Current obsolescence (o_i)	0.151^{***}	0.016^{***}	-0.421^{***}	0.085^{***}	-0.512^{***}
	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)
Current density (F_i)	-0.332^{***}	0.148^{***}	0.480^{***}	-0.336^{***}	0.013^{***}
	(0.004)	(0.005)	(0.004)	(0.004)	(0.005)
Maximum allowed density ($\bar{F}_i$)	0.235^{***}	0.974^{***}	0.062^{***}	0.253^{***}	0.058^{***}
	(0.004)	(0.0002)	(0.005)	(0.004)	(0.005)
Non-built parcel ($\mathbf{1}_{F_i=0}$)	0.918^{***}	-0.395^{***}	-0.686^{***}	0.969^{***}	
	(0.001)	(0.004)	(0.002)	(0.0003)	

Table 5: Counterfactual results

This table reports the aggregate outcomes of our counterfactual policy analyses. It reports the change of various outcomes described in columns from a counterfactual regulatory environment $\mathcal{P}$, described in the rows, to the baseline calibration $\mathcal{P}_0$. All results are computed with a 10-year horizon. In the first column, we show the change in the expected number of apartments constructed or under construction. The second column shows the same but net of demolished units. In the third and fourth columns, we separate between affordable and market-rent net expected units built. Column (5), shows the average change in expected density in percentage, with density computed as the expected number of apartments per parcel of 2,500 sqft. In Columns (6) and (7), we show the percentage change in the present value of property taxes, and total residential assets, respectively. The last three rows show the aggregate changes if the policy bundle $\mathcal{P}$ is a non-obligatory policy. In these counterfactuals, developers of each parcel select to used the policy bundle as opposed to the status-quo baseline policy $\mathcal{P}_0$ if $\pi_i(\mathcal{P}) \geq \pi_i(\mathcal{P}_0)$.

		Built environment				Public potential costs		
		$\Delta E[Apt]$ # of Apt.	$\Delta E[AptNet]$ # of Apt	$\Delta E[\text{Aff}Net]$ # of Apt.	$\Delta E[MarketNet]$ # of Apt	$\Delta E[Density]$ (%)	ΔTax (%)	ΔPV (%)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
			Unilateral policy					
(1)	*Upzoning* - $\phi = 25\%$	$86,309$	$64,589$	0	$64,589$	3.09	0.14	0.46
(2)	*Tax abatement* - $\tau = 30$	$36,869$	$19,821$	0	$19,821$	0.95	-1.16	0.29
(3)	*Affordable mandate* - $\alpha = 20\%$	$-48,931$	$-22,218$	$32,613$	$-54,831$	-1.06	-0.06	-0.42
			Mandatory Carrot & Stick Zoning					
(4)	*Density Incentive Zoning* - $\alpha = 20\%$ *&* $\phi = 25\%$	$16,169$	$26,417$	$45,633$	$-19,216$	1.26	0.04	-0.12
(5)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ *&* $\tau = 30$	$-18,278$	$-4,767$	$38,743$	$-43,510$	-0.23	-0.90	-0.21
(6)	*All* - $\alpha = 20\%$, $\phi = 25\%$, *&* $\tau = 30$	$60,793$	$54,401$	$54,558$	-157	2.60	-1.11	0.17
			Voluntary Carrot & Stick Zoning					
(7)	*Density Incentive Zoning* - $\alpha = 20\%$ *&* $\phi = 25\%$	$4,922$	$4,729$	$43,384$	$-38,654$	0.23	0.01	0.01
(8)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ *&* $\tau = 30$	$1,578$	$1,507$	$42,715$	$-41,208$	0.07	-0.14	0.01
(9)	*All* - $\alpha = 20\%$, $\phi = 25\%$, *&* $\tau = 30$	$48,081$	$41,353$	$52,015$	$-10,663$	1.98	-0.97	0.18
			Carrot & Stick Zoning with higher cost of mandate ($\epsilon_\alpha = 0.3$)					
(10)	*Density Incentive Zoning* - $\alpha = 20\%$ *&* $\phi = 25\%$	$7,252$	$20,893$	$43,850$	$-22,957$	1.00	0.02	-0.18
(11)	*Fiscal Incentive Zoning* - $\alpha = 20\%$ *&* $\tau = 30$	$-25,528$	$-8,745$	$37,293$	$-46,038$	-0.42	-0.86	-0.26
(12)	*All* - $\alpha = 20\%$, $\phi = 25\%$, *&* $\tau = 30$	$50,093$	$47,836$	$52,418$	$-4,582$	2.29	-1.06	0.10

Appendix

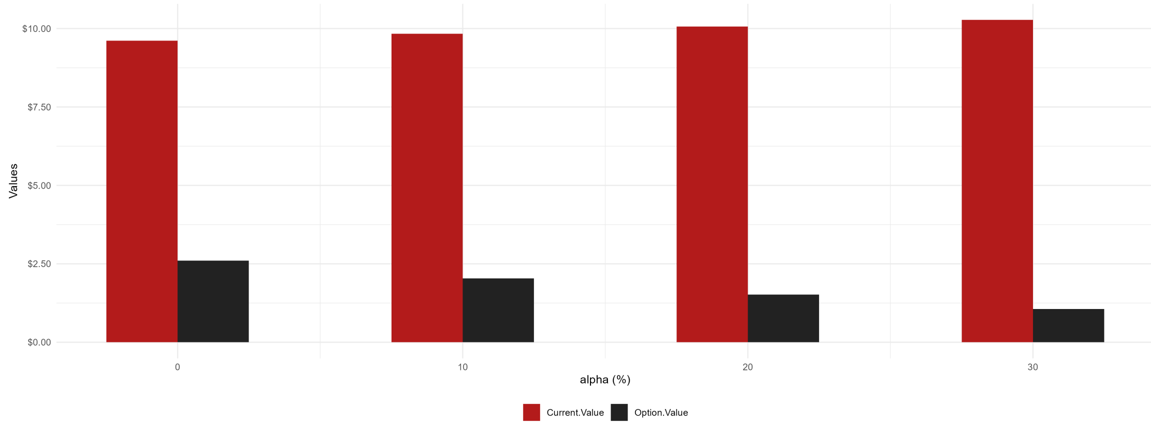
A Additional Tables & Figures

Table A1: Elasticity of real estate value with respect to the share of non-market rent units

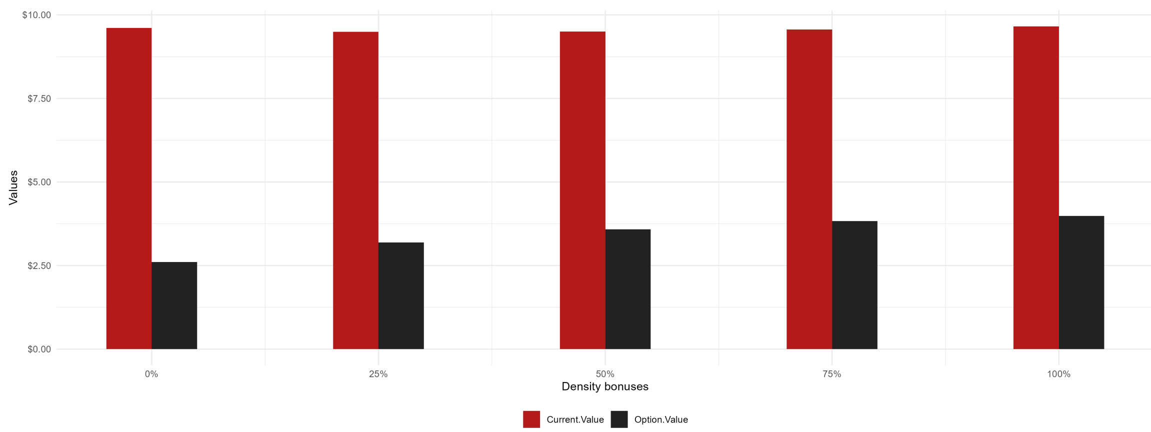
This table reports the estimates of $log(PPU)_i = \beta + \epsilon_F log(Unit_i) + \epsilon_\alpha log(1 - Aff.Share_i) + \nu_i$, *where* $log(PPU)$ *is the log of the building price divided by the number of apartment units,* $Aff.Share$ *is the share of units that are affordable identified in the data of The Department of Housing Preservation and Development (HPD). The sample consists of all building transactions from 2000 in New York City (excluding Staten Island) that contain affordable units. The time fixed effects are defined as pre-2007, 2007-2010, 2011-2015, 2016-2019, and 2020-2023. Estimates followed by ***, **, and * are statistically significant at the 1%, 5%, and 10% levels, respectively.*

	Dependent variable: log(PPU)			
	(1)	(2)	(3)	(4)
log(1-Aff.Share) - ϵ_α	0.273***	0.199***	0.198***	
	(0.057)	(0.057)	(0.043)	
... x Bronx				0.097*
				(0.058)
... x Brooklyn				0.278**
				(0.107)
... x Queens				0.372
				(0.307)
... x Manhattan				0.348***
				(0.083)
log(Units) - ϵ_F	0.074*	0.091**	0.016	0.007
	(0.042)	(0.041)	(0.031)	(0.031)
Borough Fixed effects		X	X	X
Time Fixed effects			X	X
Observations	203	203	203	203
R^2	0.114	0.196	0.568	0.583
Adjusted R^2	0.105	0.175	0.548	0.557

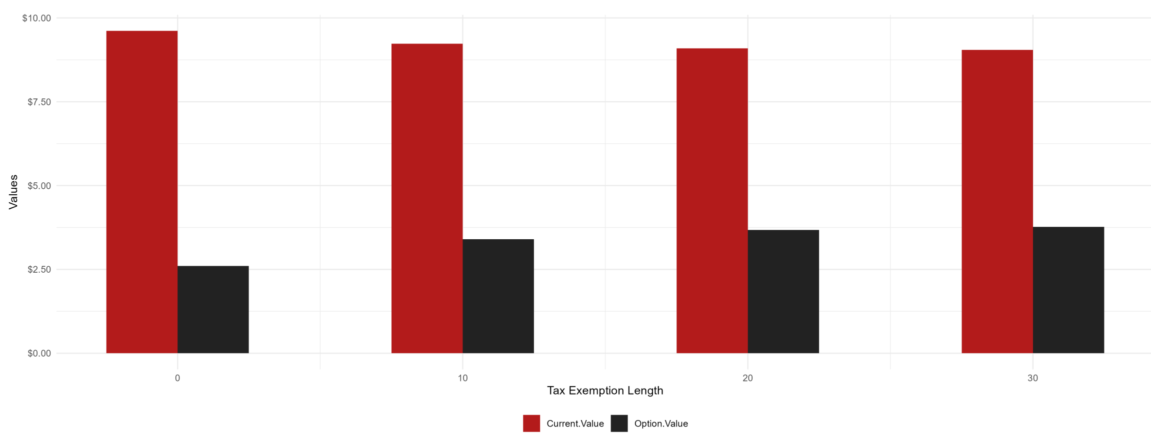
Figure A1: Policy instruments and change in option value to redevelop



A. Mandatory share for public goods (α)



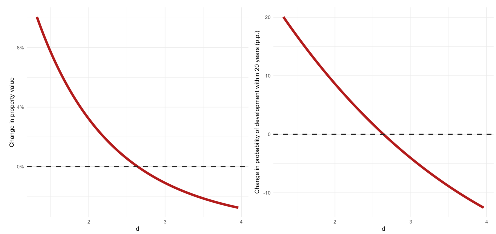
B. Density Bonuses (ϕ)



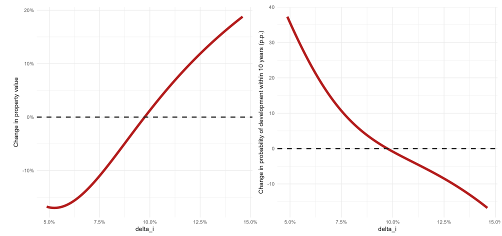
C. Length of Tax Exemptions (τ)

Note: These figures show the effects of changing the policy instruments on the present value of the current building and the present values of the redevelopment option. This option value is calculated by summing the four last elements shown in Equation (5). These figures are obtained for a parcel of land built at 25% of its capacity with $F_B = 2$ and $\bar{F} = 8$, with a rental return of $\delta_0 = .05$, and an obsolescence rate of 50% ($o_B = 0.5$). The assumption of the stochastic processes as follows: $\delta = 0.10$, $\sigma = 0.25$, $\delta_p = 0.09$, and $\sigma_p = 0.125$. All other parameters and elasticities are the baseline case shown in Table 1.

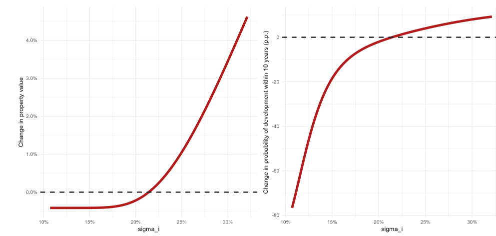
Figure A2: Marginal Effects of parameter changes



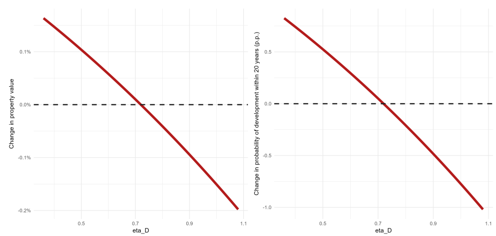
A. Construction duration – d



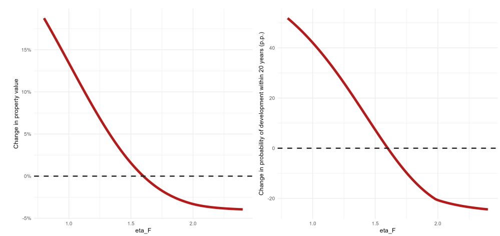
B. Return on Equity – δ_i



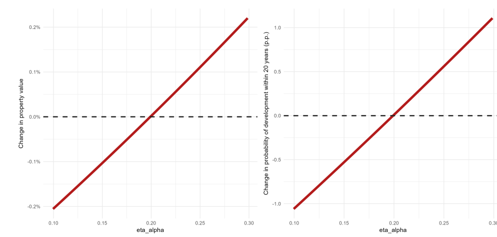
C. Volatility of Return – σ_i



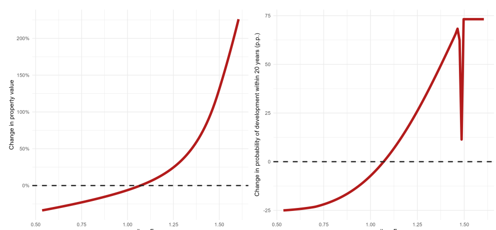
D. Height elasticity of demolition – η_D



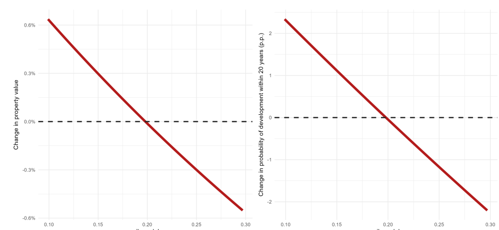
E. Height elasticity of construction – η_F



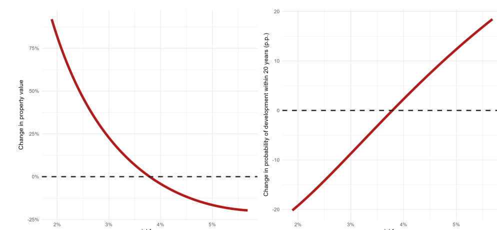
F. α elasticity of construction – η_α



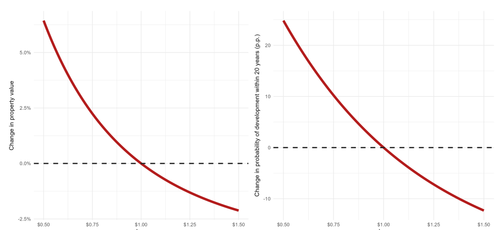
G. Height elasticity of rental value – ϵ_F



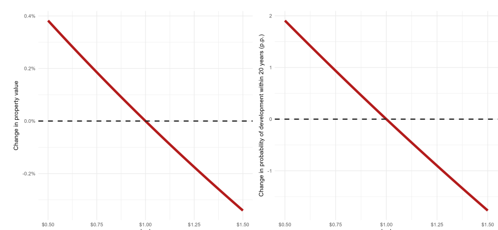
H. α elasticity of rental value – ϵ_α



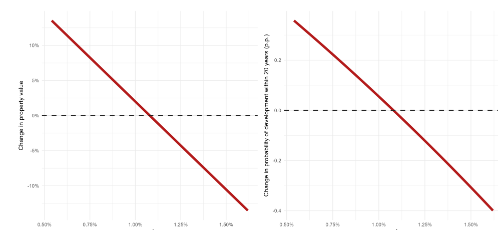
I. Risk Free Rate – r



J. Baseline construction cost – k_c



K. Baseline demolition cost – k_d



L. Property tax rate – ζ

Note: These figures show the effects of changing the baseline parameters on the change in the property value (left panels) and the change in the probability of development starting within 10 years (right panels). These figures are obtained for a parcel of land built at 40% of its capacity with $F = 2$ and $\bar{F} = 5$, with a net rental return after the obsolescence of 3% and the assumption of the stochastic processes as follows: $\delta = 0.08$, $\sigma = 0.20$, $\delta_p = 0.07$, and $\sigma_p = 0.10$. All other parameters and elasticities are the baseline case shown in Table 1. We assume a mandate to include public goods of 30% ($alpha = 0.3$) to generate comparative statics with that includes such variations.

B Proofs

B.1 Proof of Proposition 1.

Value before completed development We start the proof by working on the first integral in (5). This becomes

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_{V^e}} e^{-ru}\ (\delta_0-\psi)\ \Gamma(F_B,o_B,0)V\mathbf{1}_{V_u<V^e}\mathrm{d}u\right\} = (\delta_0-\psi)\ \Gamma(F_B,o_B,0)V\,\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} \tag{B.1}$$

with $a=(1/\sigma)\ln(V^{e*}/V)$. This last integral becomes further

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} = \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} - \mathbf{E}_{\mathbb{Q}}\left\{\int_{T_a}^{\infty} e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\}$$

Recall that $(W_t)_{t\geq 0}$ is a Brownian motion under the risk neutral measure $\mathbb{Q}$. We wish to change the probability measure to a new measure $\mathbb{P}$ so that $(Z_t)_{t\geq 0}$ becomes a Brownian motion under $\mathbb{P}$. From the equality $W_t = Z_t - bt = Z_t + \int_0^t(-b)\,\mathrm{d}s$ we can apply the Girsanov theorem and derive the appropriate density

$$\begin{aligned}\forall t\geq 0,\quad \left.\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}}\right|_{\mathcal{F}_t} &= \exp\left\{-\int_0^t(-b)\,\mathrm{d}Z_s - \frac{1}{2}\int_0^t(-b)^2\,\mathrm{d}s\right\}\\ &= \exp\left\{bZ_t - \frac{1}{2}b^2t\right\}\end{aligned} \tag{B.2}$$

Under this change of measure, we obtain

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_a} e^{-ru}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} = \mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} - \mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} \tag{B.3}$$

where $\lambda=\sqrt{2r+b^2}$

To continue, we make good use of the following result that can be found in Karatzas and Shreve (1991), p. 272.

Lemma 1 *If $\phi:\mathbb{R}\to\mathbb{R}$ is a piecewise continuous function with*

$$\int_{-\infty}^{\infty} e^{-|y|\sqrt{2\alpha}}|\phi(x+y)|\,\mathrm{d}y<\infty \quad ,\forall x\in\mathbb{R}$$

*for some constant $\alpha>0$ and $(W_t)_{t\geq 0}$ is a standard Brownian motion, the **resolvent operator** of Brownian* *motion $K_\alpha(\phi)$ is defined by*

$$K_\alpha(\phi) = \mathbf{E}\left\{\int_0^\infty e^{-\alpha t}\phi(W_t)\,\mathrm{d}t\right\} = \frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty} e^{-|y|\sqrt{2\alpha}}\phi(y)\,\mathrm{d}y$$

We obtain therefore,

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\lambda}\int_{-\infty}^{a} e^{by}e^{-|y|\lambda}\,\mathrm{d}y \\
&= \frac{-2}{b^2-\lambda^2} + \frac{e^{(b-\lambda)a}}{\lambda(b-\lambda)}
\end{aligned}$$

Additionally notice that $b^2-\lambda^2 = -2r$ and that

$$\begin{aligned}
e^{(b-\lambda)a} &= e^{(b-\lambda)\frac{1}{\sigma}\ln\left(\frac{V^e}{V}\right)} \\
&= \left(\frac{V^e}{V}\right)^{\xi}
\end{aligned} \tag{B.4}$$

with $\xi = \frac{b-\lambda}{\sigma}$.
It follows that

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^\infty e^{-ru}\mathbf{1}_{Z_u<a}\mathrm{d}u\right\} = \frac{1}{r} + \frac{1}{\lambda(b-\lambda)}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.5}$$

We now turn our attention to the second integral in (B.3). We can write

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{b(Z_u+a)}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} \\
&= e^{ba}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} \\
&= e^{(b-\lambda)a}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} \\
&= \left(\frac{V^e}{V}\right)^{\xi}\mathbf{E}_{\mathbb{P}}\left\{\int_0^\infty e^{-\frac{\lambda^2}{2}u}e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\}
\end{aligned} \tag{B.6}$$

Note that the Laplace transform of the distribution of the first passage time of a Brownian motion $(Z_t)_{t\geq 0}$ to a level a yields

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\} &= e^{-|a|\sqrt{2\lambda^2/2}} \\
&= e^{-\lambda a} \quad \text{since } a>0
\end{aligned}$$

From (**??**), it follows that

$$\begin{aligned}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u}\mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} &= \frac{1}{\lambda}\int_{-\infty}^{0} e^{by}e^{-|y|\lambda}\,\mathrm{d}y \\ &= \frac{1}{r} \end{aligned} \tag{B.7}$$

and so it follows that

$$\mathbf{E}_{\mathbb{P}}\left\{\int_{T_a}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u}\mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} = \frac{1}{r}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.8}$$

We combine results in (B.3), (**??**), (B.6), (B.4), (B.7), (**??**) and (**??**) to conclude

$$\begin{aligned}&\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{T_{V^e}} e^{-ru}\;(\delta_0-\psi)\;\Gamma(F_B,o_B,0)V\mathbf{1}_{V_u<V^e}\mathrm{d}u\right\} = \\ &\qquad\Gamma(F_B,o_B,0)\frac{(\delta_0-\psi)\;V}{r}\left[1+\left[\frac{r}{\lambda(b-\lambda)}-1\right]\left(\frac{V^e}{V}\right)^{\xi}\right]\end{aligned} \tag{B.9}$$

Demolition costs We solve the second expectation in (5) as follows:

$$\mathbf{E}_{\mathbb{Q}}\left\{e^{-rT_{V^e}}\gamma(F_B)\right\} = \gamma(F_B)\left(\frac{V^e}{V}\right)^{\xi} \tag{B.10}$$

where we use consecutively the Girsanov theorem to change to the new measure $\mathbb{P}$ delineated above and the Laplace transform of the distribution of the first passage time of a Brownian motion $(Z_t)_{t\geq 0}$ to a level a with a and ξ defined in Proposition 1.

Development costs We can now move to the third integral in (5). This is equal to

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\kappa(F,\alpha)\mathbf{1}_{V_u>V^e}\,\mathrm{d}u\right\} = \kappa(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V_e}\,\mathrm{d}u\right\}$$

we can rewrite this last integral as

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V^e}\,\mathrm{d}u\right\} = \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\,\mathrm{d}u\right\} - \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u\right\} \tag{B.11}$$

To solve the first integral in (B.11) we recall the following lemma which derives the Laplace transform for the Parisian time $H^+_{0,d}$ due to Chesney et al. (1997).

Lemma 2 *If $(W_t)_{t\geq 0}$ is a Brownian motion starting from zero, let ρ be a positive number, the Laplace transform of the first time a positive Brownian excursion lasts more than d is*

$$\mathbf{E}_{\mathbb{Q}}\left[e^{-\rho H^+_{0,d}}\right] = \frac{1}{\Phi(\sqrt{2\rho d})} \quad \textit{for all } \rho>0 \tag{B.12}$$

with

$$\Phi(x) = \int_0^{+\infty} z \exp(zx - \frac{z^2}{2})\, \mathrm{d}z = 1 + \sqrt{2\pi}x \exp\left(\frac{x^2}{2}\right)\mathcal{N}(x)$$

with $\mathcal{N}(.)$ the standard normal cumulative distribution function.

We also recall the law of $Z_{H^+_{0,d}}$. This can be inferred from Jeanblanc et al. (2009) p.249

Lemma 3 *Let $(W_t)_{t\geq 0}$ be a Brownian motion and $H^+_d(W) = \inf\{t \geq 0 : (t - g_t(W)) \geq d, W_t \geq 0\}$. The random variables H^+_d and $W_{H^+_d}$ are independent and*

$$\mathbb{P}(W_{H^+_d} \in \mathrm{d}x) = \frac{x}{d} e^{-\frac{x^2}{2d}} \mathbf{1}_{x>0} \in \mathrm{d}x \tag{B.13}$$

We proceed to solving the first integral in (B.11) with a similar line of reasoning as in the previous calculation. This yields

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru}\, \mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{\infty} e^{-ru}\, \mathrm{d}u\right\} - \mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V^e,d}}^{\infty} e^{-ru}\, \mathrm{d}u\right\} \\
&= \frac{1}{r}\left[1 - \frac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}\right]
\end{aligned} \tag{B.14}$$

The last integral in (B.11) can also be solved to obtain successively

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru} \mathbf{1}_{V_u<V^e}\, \mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{a,d}} e^{-ru} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\} \\
&= \mathbf{E}_{\mathbb{Q}}\left\{\int_0^{\infty} e^{-ru} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\} - \mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-ru} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\}
\end{aligned}$$

Using the Girsanov theorem to change the measure delineated in (B.2)

$$\begin{aligned}
\mathbf{E}_{\mathbb{Q}}\left\{\int_0^{H^+_{V^e,d}} e^{-ru} \mathbf{1}_{V_u<V^e}\, \mathrm{d}u\right\} = \mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\} \\
-\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\}
\end{aligned} \tag{B.15}$$

with $\lambda = \sqrt{2r + b^2}$. We obtain next with lemma 1,

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\, \mathrm{d}u\right\} = \frac{1}{\lambda}\int_{-\infty}^{a} e^{by} e^{-|y|\lambda}\, \mathrm{d}y$$

which after simplification becomes

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} = \frac{1}{r} + \frac{1}{\lambda(b-\lambda)}\left(\frac{V^e}{V}\right)^{\xi} \tag{B.16}$$

We can now work on the last term

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\int_{H^+_{0,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{b(Z_u+a)} \mathbf{1}_{Z_u<0}\,\mathrm{d}u\right\} \\
&= e^{ba-|a|\lambda}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}H^+_{0,d}}\int_0^{\infty} e^{bZ_{u+H^+_{0,d}}} e^{-\frac{\lambda^2}{2}u} \mathbf{1}_{Z_{u+H^+_{0,d}}<0}\,\mathrm{d}u\right\}
\end{aligned}$$

Which is equal to

$$\frac{1}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}\int_0^{\infty} \mathbb{P}(Z_{H^+_{0,d}} \in \mathrm{d}x)e^{bx}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<-x}\,\mathrm{d}u\right\} \tag{B.17}$$

but

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<-x}\,\mathrm{d}u\right\} = \frac{1}{\lambda}\int_{-\infty}^{-x} e^{by}e^{-|y|\lambda}\,\mathrm{d}y$$

since $x>0$ then $y<0$. It follows that

$$\begin{aligned}
\frac{1}{\lambda}\int_{-\infty}^{-x} e^{by}e^{-|y|\lambda}\,\mathrm{d}y &= \frac{1}{\lambda}\int_{-\infty}^{-x} e^{(b+\lambda)y}\,\mathrm{d}y \\
&= \frac{e^{-(b+\lambda)x}}{\lambda(b+\lambda)}
\end{aligned} \tag{B.18}$$

Combining (B.17) and (B.18) yields

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}\int_0^{\infty}\frac{x}{d}e^{-\frac{x^2}{2d}}e^{bx}\frac{e^{-(b+\lambda)x}}{\lambda(b+\lambda)}\,\mathrm{d}x \\
&= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{1}{\Phi(\lambda\sqrt{d})}\int_0^{\infty}\frac{x}{d}e^{-\frac{x^2}{2d}-\lambda x}\,\mathrm{d}x
\end{aligned}$$

We proceed next with a change of variables $x=\sqrt{d}y$ and conclude

$$\begin{aligned}
\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u} e^{bZ_u} \mathbf{1}_{Z_u<a}\,\mathrm{d}u\right\} &= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{1}{\Phi(\lambda\sqrt{d})}\int_0^{\infty} ye^{-\frac{y^2}{2}-\lambda\sqrt{d}y}\,\mathrm{d}y \\
&= \frac{1}{\lambda(b+\lambda)}\left(\frac{V^e}{V}\right)^{\xi}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}
\end{aligned} \tag{B.19}$$

Finally combining (B.16) and (B.19) yields the last integral from (B.11),

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u<V^e}\,\mathrm{d}u \right\} = \frac{1}{r}\left[1-\left[\frac{b+\lambda}{2\lambda}-\frac{b-\lambda}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right]\left(\frac{V^e}{V}\right)^{\xi}\right] \tag{B.20}$$

Combining (B.11), (B.14) and (B.20) yields

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_0^{H^+_{V^e,d}} e^{-ru}\mathbf{1}_{V_u>V^e}\,\mathrm{d}u \right\} = \frac{1}{r}\left[\frac{b+\lambda}{2\lambda}-\frac{b-\lambda}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}-\frac{\Phi(b\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right]\left(\frac{V^e}{V}\right)^{\xi} \tag{B.21}$$

Value after development The expectation related to the present value of the proceeds after the stopping time $H^+_{V^e,d}$ in (5) becomes

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}}^{\infty} e^{-ru}\delta\,\Gamma(F,\alpha)V_u\,\mathrm{d}u \right\} = \delta\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u \right\}$$

After the change of measure delineated in (B.2) the last expectation yields

$$\delta\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u \right\} = \delta\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u \right\}$$

We combine results from (B.6), (B.4), (B.7), (**??**) and (**??**) to solve this expectation.

$$\delta\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}u}e^{(b+\sigma)Z_u}\,\mathrm{d}u \right\} = \Gamma(F,\alpha)\frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi}V^e \tag{B.22}$$

Property taxes after τ years of tax credits The last expectation related to the present value of the property taxes after τ years of tax exemptions in (5) is solved as follows,

$$\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru}\psi\,\Gamma(F,\alpha)V_u\,\mathrm{d}u \right\} = \psi\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{a,d}+\tau}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u \right\}$$

After the change of measure delineated in (B.2) the last expectation becomes

$$\psi\,\Gamma(F,\alpha)\mathbf{E}_{\mathbb{Q}}\left\{ \int_{H^+_{a,d}+\tau}^{\infty} e^{-ru}Ve^{\sigma Z_u}\,\mathrm{d}u \right\} = \psi\,\Gamma(F,\alpha)V\,\mathbf{E}_{\mathbb{P}}\left\{ \int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)}e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u \right\}$$

Thanks to the equality in law between $H^+_{a,d}$ and $H^+_{0,d}+T_a$ for two independent copies, the independence of T_a and $H^+_{0,d}$, the Laplace transform of the distribution of the first passage time of a

Brownian motion $(Z_t)_{t\geq 0}$ to a level a, and the Independence between $H^+_{0,d}$ and $Z_{H^+_{0,d}}$ we can write

$$\mathbf{E}_{\mathbb{P}}\left\{\int_{H^+_{a,d}}^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)} e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} = \\ e^{(b+\sigma)a}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}T_a}\right\}\mathbf{E}_{\mathbb{P}}\left\{e^{-\frac{\lambda^2}{2}H^+_{0,d}}\right\}\mathbf{E}_{\mathbb{P}}\left\{e^{(b+\sigma)Z_{H^+_{0,d}}}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)} e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\}$$

and the last integral becomes

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)} e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} = e^{-\lambda^2\frac{\tau}{2}}\mathbf{E}_{\mathbb{P}}\left\{e^{(b+\sigma)Z_\tau}\right\}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\}$$

Z_τ is a Brownian motion under $\mathbb{P}$ distributed $\mathcal{N}(0,\tau)$. We recall the following result from Borodin and Salminen (2015) p. 153

Lemma 4 *Let $(W_t)_{t\geq 0}$ be a Brownian motion. The expectation of $e^{\beta W_t}$ at time T with the process $(W_t)_{t\geq 0}$ started at time x is*

$$\mathbf{E}_x\left\{e^{\beta W_T}\right\} = e^{\beta x - \frac{\beta^2 T}{2}}$$

it follows that

$$\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}(u+\tau)} e^{(b+\sigma)Z_{u+\tau}}\,\mathrm{d}u\right\} = e^{-\lambda^2\frac{\tau}{2}} e^{-(b+\sigma)^2\frac{\tau}{2}}\mathbf{E}_{\mathbb{P}}\left\{\int_0^{\infty} e^{-\frac{\lambda^2}{2}u} e^{(b+\sigma)Z_u}\,\mathrm{d}u\right\}$$

Combining result from (B.6), (B.4), (B.7), (**??**) and (**??**) yields

$$\mathbf{E}_{\mathbb{Q}}\left\{\int_{H^+_{V^e,d}+\tau}^{\infty} e^{-ru}\psi\,\Gamma(F,\alpha)V_u\,\mathrm{d}u\right\} = \Gamma(F,\alpha)\frac{\psi h}{\delta}\frac{\Phi\left((\sigma+b)\sqrt{d}\right)}{\Phi(\lambda\sqrt{d})}\left(\frac{V^e}{V}\right)^{\xi} V^e$$

with $h = e^{-\left[(b+\sigma)^2+\lambda^2\right]\frac{\tau}{2}}$. This concludes the proof.

B.2 Proof of Proposition 2.

We start by rearranging the property value in equation (6) in the following way:

$$\pi(V,V^e,F_B,F,\alpha,\phi,\tau;\Theta) = \Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V + \Bigg[\Gamma(F_B,o_B,0)\frac{\delta_0-\psi}{r}V\left[\frac{r}{\lambda(b-\lambda)}-1\right] \\ + \vartheta V^e - \gamma(F_B) - \frac{\kappa(F,\alpha)}{r}\Big(B(d)-A(d)\Big)\Bigg]\left(\frac{V^e}{V}\right)^{\xi}$$

with $\vartheta = \Gamma(F,0,\alpha)\frac{\delta-\psi h}{\delta}C(d)$.

We establish the expansion threshold under regulatory environment $\mathcal{P}$, denoted as V^{e*}, as the parameter that optimizes the property value. In other words,

$$V^{e*} = \arg\max \pi(V, V^e, F_B, F, \alpha, \phi, \tau; \Theta)$$

It follows that V^{e*} must satisfy the first order conditions and the second order conditions. The partial derivative of $\pi(V, V^e, F_B, F, \alpha, \phi, \tau; \Theta)$ with respect to V^e is equal to

$$\frac{\partial \pi(V, V^e, F_B, F, \alpha, \phi, \tau; \Theta)}{\partial V^e} = \vartheta \left(\frac{V^e}{V}\right)^{\xi} + \frac{\xi}{V}\Bigg[\Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right] \\ + \vartheta V^e - \gamma(F_B) - \frac{\kappa(F, \alpha)}{r}\Big(B(d) - A(d)\Big)\Bigg]\left(\frac{V^e}{V}\right)^{\xi - 1}$$

Which can be simplified to

$$\left(\frac{V^e}{V}\right)^{\xi}\Bigg[\vartheta + \frac{\xi}{V^e}\Bigg[\Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right] \\ + \vartheta V^e - \gamma(F_B) - \frac{\kappa(F, \alpha)}{r}\Big(B(d) - A(d)\Big)\Bigg]$$

We know that $V^e > V$ so $V^e \neq 0$. This implies that first order conditions are satisfied if and only if

$$\vartheta + \frac{\xi}{V^e}\Bigg[\Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right] + \vartheta V^e - \gamma(F_B) - \frac{\kappa(F, \alpha)}{r}\Big(B(d) - A(d)\Big) = 0$$

some further simplification yields V^{e*}.

$$V^{e*} = \frac{\xi}{1+\xi}\frac{1}{\vartheta}\Bigg[\gamma(F_B) + \frac{\kappa(F, \alpha)}{r}\Big(B(d) - A(d)\Big) - \Gamma(F_B, o_B, 0)\frac{\delta_0 - \psi}{r}V\left[\frac{r}{\lambda(b-\lambda)} - 1\right]\Bigg]$$

B.3 Proof of Proposition 3.

Recall that $V_t = Ve^{\sigma Z_t}$, where $Z_t = bt + W_t$, and $b = \frac{1}{\sigma}(r - \delta - \frac{\sigma^2}{2})$. This implies that $T^{V^{e*}}(V) = \inf\{t \geq 0 : Z_t = a\}$ with $a = (1/\sigma)\ln(V^{e*}/V)$. Recall that $(W_t)_{t\geq 0}$ is a Brownian motion under the risk neutral measure $\mathbb{Q}$ and that after we change the probability measure to a new measure $\mathbb{P}$, $(Z_t)_{t\geq 0}$ becomes a Brownian motion under $\mathbb{P}$. We recall the law of the hitting time $T^{V^{e*}}$ of the level V^{e*} from Jeanblanc et al. (2009) p.140

Lemma 5 *Let W_t be a Brownian motion and for any $x > 0$ define $T_x(W) = \inf\{t \geq 0 | W_t = x\}$. The density of the random variable T_x is given by* $\mathbf{P}(T_x \in \mathrm{d}t) = \frac{x}{\sqrt{2\pi t^3}}\exp\left(-\frac{x^2}{2t}\right)\mathrm{d}t$

We use Lemma 5 to conlude the proof.